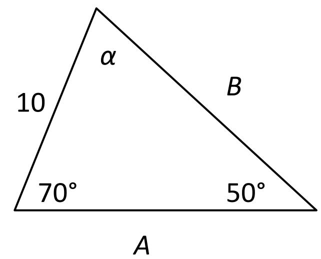
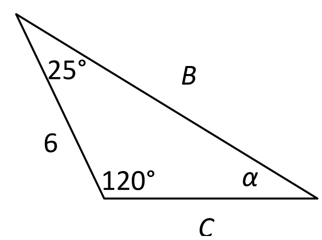
8.1 Solutions to Exercises



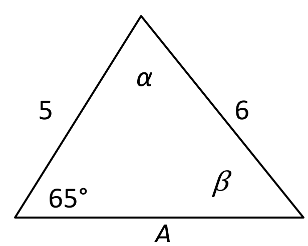
1. Since the sum of all angles in a triangle is 180°, 180° = 70° + 50° + α. Thus α = 60°.

The easiest way to find A and B is to use Law of Sines. According to Law of Sines, $\frac{\sin(\alpha)}{a}=\frac{\sin(\beta)}{b}=\frac{\sin(\gamma)}{c}$, where each angle is across from its respective side.

Thus, $\frac{\sin(60)}{A}=\frac{\sin(70)}{B}=\frac{\sin(50)}{10}$. Then $B = \frac{10\sin(70)}{\sin(50)} \approx 12.26$, and $A = \frac{10\sin(60)}{\sin(50)} \approx 11.31$.



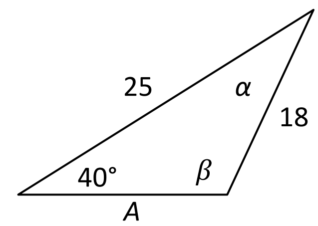
3. Since the sum of all angles in a triangle is 180°, 180° = 25° + 120° + α. Thus α = 35°. The easiest way to find C and B is to use Law of Sines. According to Law of Sines, $\frac{\sin(\alpha)}{a}=\frac{\sin(\beta)}{b}=\frac{\sin(\gamma)}{c}$, where each angle is across from its respective side. Thus, $\frac{\sin(35)}{6}=\frac{\sin(120)}{B}=\frac{\sin(25)}{C}$. Then $B = \frac{6\sin(120)}{\sin(35)} \approx 9.06$, and $C = \frac{6\sin(25)}{\sin(35)} \approx 4.42$.



5. According to Law of Sines, $\frac{\sin(\alpha)}{a}=\frac{\sin(\beta)}{b}=\frac{\sin(\gamma)}{c}$, where each angle is across from its respective side. Thus, $\frac{\sin(\alpha)}{A}=\frac{\sin(\beta)}{5}=\frac{\sin(65)}{6}$. Thus $\frac{\sin(\beta)}{5}=\frac{\sin(65)}{6}$ and $\beta = \sin^{-1}\left(\frac{5\sin(65)}{6}\right) \approx 49.05°$. Recall there are two possible solutions from 0 to 2π; to find the other solution use symmetry. β could also be 180 - 49.05 = 130.95. However, when this and the given side are added together, their sum is greater than 180, so 130.95 cannot be β. Since the sum of all angles in a triangle is 180°, 180° = 49.05° + 65° + α. Thus α = 65.95°.

Again from Law of Sines $\frac{\sin(65.95)}{A} = \frac{\sin(49.05)}{5}$, so $A = \frac{5\sin(65.95)}{\sin(49.05)} \approx 6.05$.

7. According to Law of Sines, $\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$,



where each angle is across from its respective side. Thus, $\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{5} = \frac{\sin(65)}{6}$. Thus $\frac{\sin(\beta)}{25} = \frac{\sin(40)}{18}$ and $\beta = \sin^{-1}\left(\frac{25\sin(40)}{18}\right) \approx 63.33°$. Whenever solving for angles with the Law of Sines there are two possible solutions. Using symmetry the other solution may be 180 - 63.33 = 116.67. However, the triangle shown has an obtuse angle β so $\beta = 116.67°$. Since the sum of all angles in a triangle is 180°, $180° = 116.78° + 40° + \alpha$. Thus $\alpha = 23.22°$.

Again from Law of Sines $\frac{\sin(23.22)}{A} = \frac{\sin(40)}{18}$, so $A = \frac{18\sin(23.22)}{\sin(40)} \approx 11.042$.

9. Since the sum of all angles in a triangle is 180°, $180° = 69° + 43° + \beta$. Thus $\beta = 68°$. Using Law of Sines, $\frac{\sin(43)}{a} = \frac{\sin(68)}{20} = \frac{\sin(69)}{c}$, so $a = \frac{20\sin(43)}{\sin(68)} \approx 14.71$ and $c = \frac{20\sin(69)}{\sin(68)} \approx 20.13$.

Once two sides are found the third side can be found using Law of Cosines. If side *c* is found first then side *a* can be found using, $a^2 = c^2 + b^2 - 2bc\cos(\alpha) = 20.13^2 + 20^2 - 2(20)(20.13)\cos(43)$

11. To find the second angle, Law of Sines must be used. According to Law of Sines $\frac{\sin(119)}{26} = \frac{\sin(\beta)}{14}$, so $\beta = \sin^{-1}\left(\frac{14}{26}\sin(119)\right) \approx 28.10$. Whenever the inverse sine function is used there are two solutions. Using symmetry the other solution could be 180 - 20.1 = 159.9, but since that number added to our given angle is more than 180, that is not a possible solution. Since the sum of all angles in a triangle is 180°, $180° = 119° + 28.10° + \gamma$. Thus $\gamma = 32.90°$. To find the last side either Law of Sines or Law of Cosines can be used. Using Law of Sines, $\frac{\sin(119)}{26} = \frac{\sin(28.10)}{14} = \frac{\sin(32.90)}{c}$, so $c = \frac{14\sin(32.90)}{\sin(28.10)} = \frac{26\sin(32.90)}{\sin(119)} \approx 16.15$. Using Law of Cosines, $c^2 = a^2 + b^2 - 2ab\cos(\gamma) = 26^2 + 14^2 - 2(26)(14)\cos(32.90)\ so\ c \approx 16.15$.

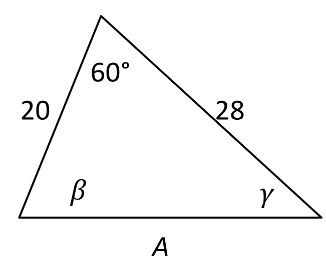
13. To find the second angle, Law of Sines must be used. According to Law of Sines, $\frac{\sin(50)}{45} = \frac{\sin(\alpha)}{105}$. However, when solved the quantity inside the inverse sine is greater than 1, which is out of the range of sine, and therefore out of the domain of inverse sine, so it cannot be solved.

15. To find the second angle Law of Sines must be used. According to Law of Sines, $\frac{\sin(43.1)}{184.2} = \frac{\sin(\beta)}{242.8}$, so $\beta = \sin^{-1}\left(\frac{242.8}{184.2}\sin(43.1)\right) \approx 64.24$ or $\beta = 180 - 64.24 = 115.76$. Since the sum of all angles in a triangle is 180°, $180° = 43.1° + 64.24° + \gamma$ or

$180° = 43.1° + 115.76° + \gamma$. Thus $\gamma = 72.66°$ or $\gamma = 21.14°$. To find the last side either Law of Sines or Law of Cosines can be used. Using Law of Sines, $\frac{\sin(43.1)}{184.2} = \frac{\sin(64.24)}{242.8} = \frac{\sin(72.66)}{c}$, so $c = \frac{184.2\sin(72.66)}{\sin(43.1)} = \frac{242.8\sin(72.66)}{\sin(64.24)} \approx 257.33$. The same procedure can be used to find the alternate solution where $c = 97.238$.

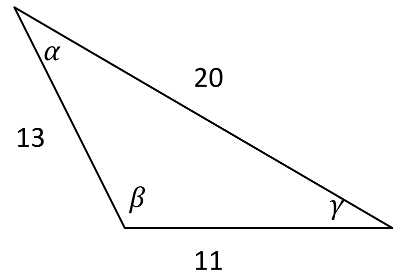
Using Law of Cosines, $c^2 = a^2 + b^2 - 2ab\cos(\gamma) = 184.2^2 + 242.8^2 - 2(184.2)(242.8)\cos(72.66)$ so $c \approx 257.33$.

17. Because the givens are an angle and the two sides around it, it is best to use Law of Cosines to find the third side. According to Law of Cosines, $a^2 = c^2 + b^2 - 2bc\cos(\alpha) = 20^2 + 28^2 - 2(20)(28)\cos(60)$, so $A \approx 24.98$.



Using Law of Sines, $\frac{\sin(60)}{24.98} = \frac{\sin(\beta)}{28} = \frac{\sin(\gamma)}{20}$, so $\beta = \sin^{-1}\left(\frac{28}{24.98}\sin(60)\right) \approx 76.10$. Because the angle β in the picture is acute, it is not necessary to find the second solution. The sum of the angles in a triangle is 180° so $180° = 76.10° + 60° + \gamma$ and $\gamma = 43.90°$.

19. In this triangle only sides are given, so Law of Sines cannot be used. According to Law of Cosines:



$a^2 = c^2 + b^2 - 2bc\cos(\alpha)$ so $11^2 = 13^2 + 20^2 - 2(13)(20)\cos(\alpha)$. Then $\alpha = \cos^{-1}\left(\frac{13^2+20^2-11^2}{2(13)(20)}\right) \approx 30.51$.

To find the next sides Law of Cosines or Law of Sines can be used. Using Law of Cosines, $b^2 = c^2 + a^2 - 2ac\cos(\beta) \Rightarrow 20^2 = 13^2 + 11^2 - 2(13)(11)\cos(\beta)$ so $\beta = \cos^{-1}\left(\frac{13^2+11^2-20^2}{2(13)(11)}\right) \approx 112.62$.

The sum of the angles in a triangle is 180 so 180° = 112.62° + 30.51° + γ and γ = 36.87°.

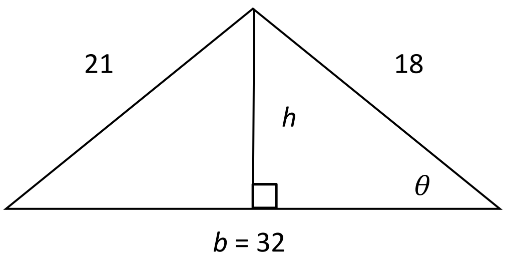
21. Because the angle corresponds to neither of the given sides it is easiest to first use Law of Cosines to find the third side. According to Law of Cosines, $c^2 = a^2 + b^2 - 2ab\cos(\gamma) = 2.49^2 + 3.13^2 - 2(2.49)(3.13)\cos(41.2)$, so $c = 2.07$.

To find the α or β either Law of Cosines or Law of Sines can be used. Using Law of Sines, $\frac{\sin(41.2)}{2.07} = \frac{\sin(\alpha)}{2.49}$, so $\alpha = \sin^{-1}\left(\frac{2.49\sin(41.2)}{2.07}\right) \approx 52.55°$. The inverse sine function gives two solutions so α could also be 180 - 52.55 = 127.45. However, side b is larger than side c so angle γ must be smaller than β, which could not be true if α = 127.45, so α = 52.55. The sum of angles in a triangle is 180°, so 180° = 52.55 + 41.2 + β and β = 86.26°.

23. Because the angle corresponds to neither of the given sides it is easiest to first use Law of Cosines to find the third side. According to Law of Cosines, $a^2 = c^2 + b^2 - 2cb\cos(\alpha) = 7^2 + 6^2 - 2(7)(6)\cos(120)$, so $a = 11.27$.

Either Law of Cosines or Law of Sines can be used to find β and γ. Using Law of Cosines, $b^2 = c^2 + a^2 - 2ac\cos(\beta) \Rightarrow 6^2 = 7^2 + 11.27^2 - 2(7)(11.27)\cos(\beta)$, so $\beta = \cos^{-1}\left(\frac{7^2+11.27^2-6^2}{2(7)(11.27)}\right) \approx 27.46$. The sum of all the angles in a triangle is 180° so 180° = 27.46° + 120° + γ, and γ = 32.54°.

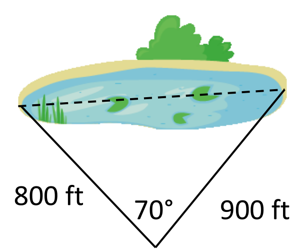
25. The equation of the area of a triangle is $A = \frac{1}{2}bh$. It is important to draw a picture of the triangle to figure out which angles and sides need to be found.



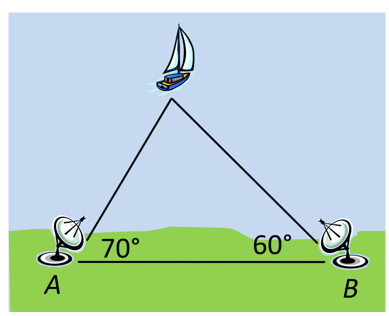
With the orientation chosen the base is 32. Because the height makes a right triangle inside of the original triangle, all that needs to be found is one angle to find the height, using trig. Either side of the triangle can be used.

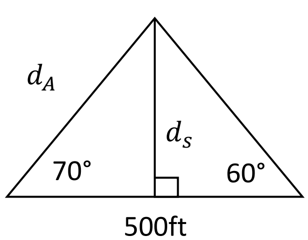
According to Law of Cosines $21^2 = 18^2 + 32^2 - 2(18)(32)\cos(\theta)$ so, $\theta = \cos^{-1}\left(\frac{18^2+32^2-21^2}{2(18)(32)}\right) \approx 38.06$. Using this angle, $h = \sin(38.06)18 \approx 11.10$, so $A = \frac{32 \cdot 11.10}{2} \approx 177.56$.

27. Because the angle corresponds to neither of the given sides it is easiest to first use Law of Cosines, to find the third side. According to Law of Cosines $d^2 = 800^2 + 900^2 - 2(800)(900)\cos(70)$ where d is the distance across the lake. $d = 978.51$ft



29. To completely understand the situation it is important to first draw a new triangle, where d_s is the distance from the boat to the shore and d_A is the distance from station A to the boat.



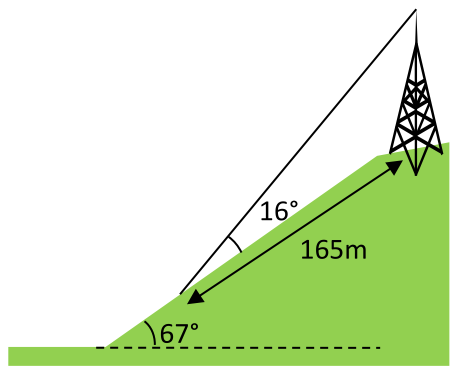


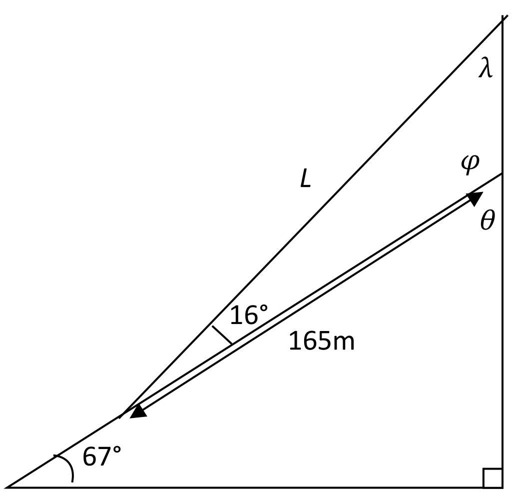
Since the only side length given does not have a corresponding angle given, the corresponding angle (θ) must first be found. The sum of all angles in a triangle must be 180°, so $180° = 70° + 60° + \theta, and\ \theta = 50°$.

Knowing this angle allows us to use Law of Sines to find d_A. According to Law of Sines $\frac{\sin(50)}{500} = \frac{\sin(60)}{d_A} \Rightarrow d_A = \frac{500\sin(60)}{\sin(50)} \approx 565.26ft.$

To find d_s, trigonometry of the left hand right triangle can be used. $d_A = \sin(70)565.26ft \approx 531.17ft.$

31. The hill can be visualized as a right triangle below the triangle that the wire makes, assuming that, a line perpendicular to the base of 67° angle is dropped from the top of the tower. Let L be the length of the guy wire.





In order to find L using Law of Cosines, the height of the tower, and the angle between the tower and the hill need to be found.

To solve using Law of Sines only the angle between the guy wire and the tower, and the angle corresponding with L need to be found.

Since finding the angles requires only basic triangle relationships, solving with Law of Sines will be the simpler solution.

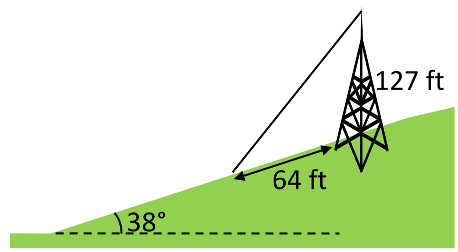
The sum of all angles in a triangle is 180°, this rule can be used to find the angle of the hill at the tower location (θ). $180° = 90° + 67° + \theta, so\ \theta = 23°.$

θ and the angle between the tower and the hill (φ) are supplementary angles, so $\theta + \varphi = 180°$, thus $\varphi = 157°$.

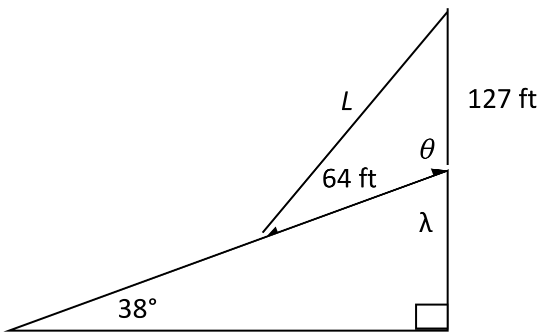
Using once again the sum of all angles in a triangle, $\varphi + 16 + \lambda = 180,\ so\ \lambda = 7°$.

According to Law of Sines, $\frac{\sin(7)}{165} = \frac{\sin(157)}{L}$, so $L = \frac{165\sin(157)}{\sin(7)} \approx 529.01\ m.$

33. Let L be the length of the wire.



The hill can be visualized as a right triangle, assuming that a line perpendicular to the base of the 38° angle is dropped from the top of the tower.

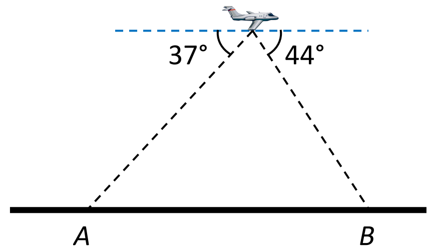


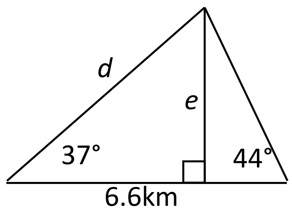
Because two sides of the triangle are given, and the last side is what is asked for, it is best to use Law of Cosines. In order to use Law of Cosines, the angle θ corresponding to L needs to be found. In order to find θ the last angle in the right triangle (λ) needs to be found.

The sum of all angles in a triangle is 180°, so 180° = 38° + 90° + λ, and λ = 52°. λ and θ are suplimentary angles so $\lambda + \theta = 180°$, and θ = 128°.

According to Law of Cosines, $L^2 = 127^2 + 64^2 - 2(127)(64)\cos(128)$, so $L = 173.88ft.$

35. Using the relationship between alternate interior angles, the angle at A inside the triangle is 37°, and the angle at B inside the triangle is 44°.





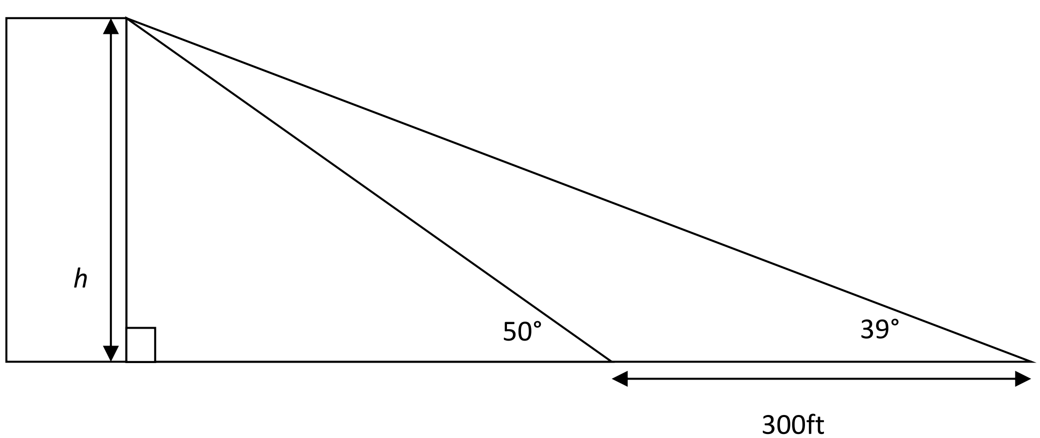
Let e be the elevation of the plane, and let d be the distance from the plane to point A. To find the last angle (θ), use the sum of angles. $180° = 37° + 44° + \theta$, so $\theta = 99°$.

Because there is only one side given, it is best to use Law of Sines to solve for d. According to Law of Sines, $\frac{\sin(44)}{d} = \frac{\sin(99)}{6.6}$, so $d = \frac{6.6\sin(44)}{\sin(99)} \approx 4.64$ km.

Using the right triangle created by drawing e and trigonometry of that triangle, e can be found. $e = 4.64\sin(37) \approx 2.79$ km.

37. Assuming the building is perpendicular with the ground, this situation can be drawn as two triangles.

Let h = the height of the building. Let x = the distance from the first measurement to the top of the building.



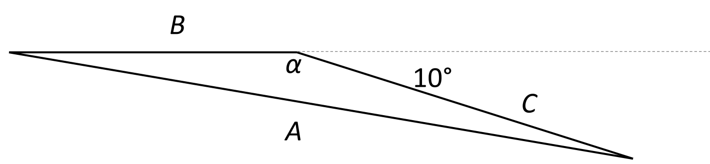
In order to find *h*, we need to first know the length of one of the other sides of the triangle. *x* can be found using Law of Sines and the triangle on the right.

The angle that is adjacent to the angle measuring 50° has a measure of 130°, because it is supplementary to the 50° angle. The angle of the top of the right hand triangle measures 11° since all the angles in the triangle have a sum of 180°.

According to Law of Sines, $\frac{\sin(39)}{x} = \frac{\sin(11)}{300ft}$, so $x = 989.45$ft.

Finding the value of h only requires trigonometry. $h = (989.45 \text{ ft})\sin(50) \approx 757.96$ ft.

39. Because the given information tells us two sides and information relating to the angle opposite the side we need to find, Law of Cosines must be used.



The angle α is supplementary with the 10° angle, so $\alpha = 180° - 10° = 170°$.

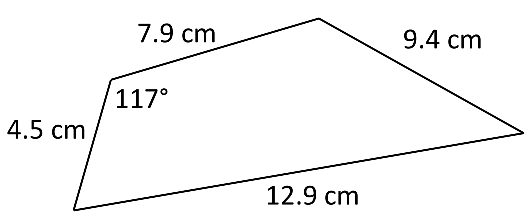
From the given information, the side lengths can be found:

$B = 1.5 \text{ hours} \cdot \frac{680 \text{ miles}}{1 \text{ hour}} = 1020 \text{ miles}$.

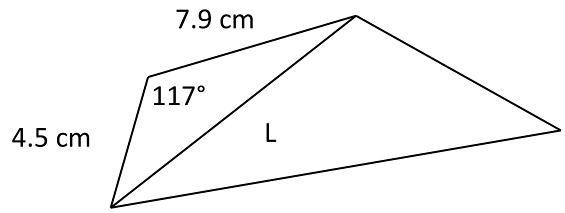
$C = 2 \text{ hours} \cdot \frac{680 \text{ miles}}{1 \text{ hour}} = 1360 \text{ miles}$.

According to Law of Cosines: so $A^2 = (1020)^2 + (1360\ )^2 - 2(1020)(1360\ )\cos(170)$. Solving for *A* gives $A \approx 2{,}371.13$ miles.

41. Visualized, the shape described looks like:

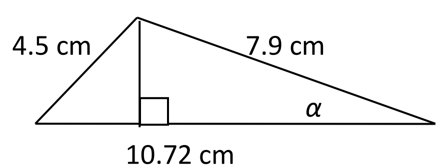


Drawing a line from the top right corner to the bottom left corner breaks the shape into two triangles. Let L be the length of the new line.



Because the givens are two sides and one angle, Law of Cosines can be used to find length L.
$L^2 = 4.5^2 + 7.9^2 - 2(4.5)(7.9)\cos(117)$ L=10.72.

The equation for the area of a triangle is $A = \frac{1}{2}bh$. To find the area of the quadrilateral, it can be broken into two separate triangles, with their areas added together.

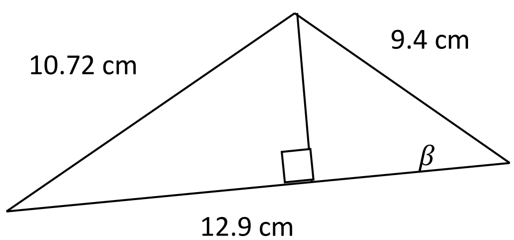


In order to use trig to find the area of the first triangle, one of the angles adjacent to the base must be found, because that angle will be the angle used in the right triangle to find the height of the right triangle (h).

According to Law of Cosines $4.5^2 = 10.72^2 + 7.9^2 - 2(10.72)(7.9)\cos(\alpha)$, so $\alpha \approx 21.97°$.

Using trigonometry, $h = 7.9\sin(21.97) \approx 2.96$ cm. So, $A_1 = \frac{(2.96cm)(10.72cm)}{2} \approx 15.84\ cm^2$.

The same procedure can be used to evaluate the Area of the second triangle.



According to Law of Cosines $10.72^2 = 9.4^2 + 12.9^2 - 2(9.4)(12.9)\cos(\beta)$, so $\beta \approx 54.78°$.

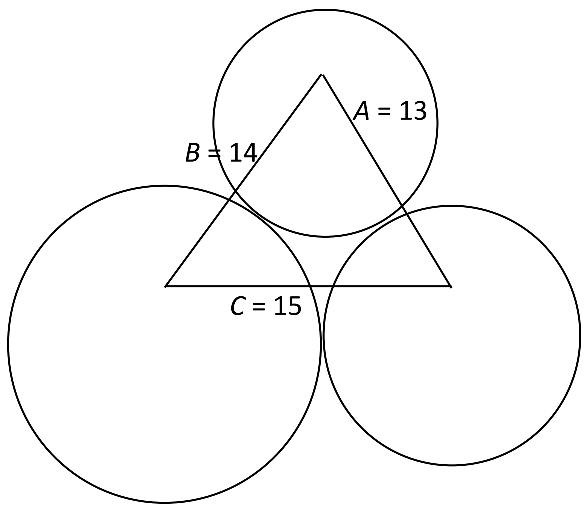
Using trigonometry, $h = 9.4\sin(54.78) \approx 7.68$ cm.

So, $A_2 = \frac{(7.68cm)(12.9cm)}{2} \approx 49.53\ cm^2$.

The area of the quadrilateral is the sum of the two triangle areas so, $A_q = 49.53 + 15.84 = 65.37\ \text{cm}^2$.

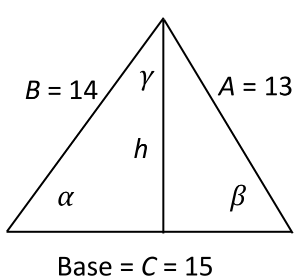
41. If all the centers of the circles are connected a triangle forms whose sides can be found using the radii of the circles.

Let side *A* be the side formed from the 6 and 7 radii connected. Let side *B* be the side formed from the 6 and 8 radii connected. Let side *C* be the side formed by the 7 and 8 radii connected.



In order to find the area of the shaded region we must first find the area of the triangle and the areas of the three circle sections and find their difference.

To find the area of the triangle the height must be found using trigonometry and an angle found using Law of Cosines



According to Law of Cosines, $13^2 = 14^2 + 15^2 - 2(14)(15)\cos(\alpha)\,, so\ \alpha \approx 53.13°$.

Using trigonometry $h = 14\sin(53.13°) = 11.2$, so $A_T = \frac{(11.2)(15)}{2} = 84$.

To find the areas of the circle sections, first find the areas of the whole circles. The three areas are, $A_6 = \pi(6)^2 \approx 113.10$, $A_7 = \pi(7)^2 \approx 153.94$, and $A_8 = \pi(8)^2 \approx 201.06$.

To find the Area of the portion of the circle, set up an equation involving ratios.
$\frac{\text{Section Area }(As)}{\text{Circle Area }(Ac)} = \frac{\text{Section angle }(Ds)}{\text{Circle Angle }(360)} \Rightarrow As = (Ac)\frac{Ds}{360}$.

The section angles can be found using the original triangle and Law of Cosines.

$14^2 = 13^2 + 15^2 - 2(13)(15)\cos(\beta)$, so $\beta \approx 59.49°$.

The sum of all angles in a triangle has to equal 180°, so $180° = \alpha + \beta + \gamma = 59.49° + 53.13° + \gamma, so, \gamma = 67.38°$.

Using this information, $A_{s6} = \frac{(67.38)(113.10)}{360} \approx 21.17$, $A_{s7} = \frac{(59.49)(153.94)}{360} \approx 25.44$, $A_{s8} = \frac{(53.13)(201.06)}{360} \approx 29.67$. So, the area of the shaded region $A_f = A_T - A_{s6} - A_{s7} - A_{s8} = 84 - 21.17 - 25.44 - 29.67 = 7.72$.

8.2 Solutions to Exercises

1. The Cartesian coordinates are $(x, y) = (r\cos(\theta), r\sin(\theta))$
$$= \left(7\cos\left(\frac{7\pi}{6}\right), 7\sin\left(\frac{7\pi}{6}\right)\right) = \left(-7\cos\left(\frac{\pi}{6}\right), -7\sin\left(\frac{\pi}{6}\right)\right) = \left(-\frac{7\sqrt{3}}{2}, -\frac{7}{2}\right)$$

3. The Cartesian coordinates are $(x, y) = (r\cos(\theta), r\sin(\theta))$
$$= \left(4\cos\left(\frac{7\pi}{4}\right), 4\sin\left(\frac{7\pi}{4}\right)\right) = \left(4\cos\left(\frac{\pi}{4}\right), -4\sin\left(\frac{\pi}{4}\right)\right) = \left(\frac{4\sqrt{2}}{2}, -\frac{4\sqrt{2}}{2}\right) = (2\sqrt{2}, -2\sqrt{2})$$

5. The Cartesian coordinates are $(x, y) = (r\cos(\theta), r\sin(\theta))$

$= \left(6\cos\left(-\frac{\pi}{4}\right), 6\sin\left(-\frac{\pi}{4}\right)\right) = \left(6\cos\left(\frac{\pi}{4}\right), -6\sin\left(\frac{\pi}{4}\right)\right) = \left(\frac{6\sqrt{2}}{2}, -\frac{6\sqrt{2}}{2}\right) = (3\sqrt{2}, -3\sqrt{2})$

7. The Cartesian coordinates are $(x, y) = (r\cos(\theta), r\sin(\theta)) = \left(3\cos\left(\frac{\pi}{2}\right), 3\sin\left(\frac{\pi}{2}\right)\right) = (0, 3)$

9. The Cartesian coordinates are $(x, y) = (r\cos(\theta), r\sin(\theta)) = \left(-3\cos\left(\frac{\pi}{6}\right), -3\sin\left(\frac{\pi}{6}\right)\right) = \left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

11. The Cartesian coordinates are $(x, y) = (r\cos(\theta), r\sin(\theta)) = (3\cos(2), 3\sin(2)) \approx (-1.2484, 2.7279)$

13. $(4, 2) = (x, y) = (r\cos(\theta), r\sin(\theta))$. Then $\tan(\theta) = \frac{y}{x} = \frac{2}{4} = \frac{1}{2}$. Since (x, y) is located in the first quadrant, where $0 \le \theta \le \frac{\pi}{2}$, $\theta = \tan^{-1}\left(\frac{1}{2}\right) \approx 0.46365$. And $r^2 = x^2 + y^2 = 4^2 + 2^2 = 20 \rightarrow r = \sqrt{20} = 2\sqrt{5}$.

15. $(-4, 6) = (x, y) = (r\cos(\theta), r\sin(\theta))$. Then $\tan(\theta) = \frac{y}{x} = \frac{6}{-4} = -\frac{3}{2}$. Since (x, y) is located in the second quadrant, where $\frac{\pi}{2} \le \theta \le \pi$, and $\tan(\theta) = \tan(\theta + \pi)$, $\theta = \tan^{-1}\left(-\frac{3}{2}\right) + \pi \approx -0.9828 + \pi \approx 2.1588$. And $r^2 = x^2 + y^2 = (-4)^2 + 6^2 = 52 \rightarrow r = 2\sqrt{13}$.

17. $(3, -5) = (x, y) = (r\cos(\theta), r\sin(\theta))$. Then $\tan(\theta) = \frac{y}{x} = \frac{-5}{3}$. Since (x, y) is located in the fourth quadrant, where $\frac{3\pi}{2} \le \theta \le 2\pi$, $\theta = \tan^{-1}(-\frac{5}{3}) + 2\pi \approx -1.0304 + 2\pi \approx 5.2528$. And $r^2 = x^2 + y^2 = 3^2 + (-5)^2 = 34 \rightarrow r = \sqrt{34}$.

19. $(-10, -13) = (x, y) = (r\cos(\theta), r\sin(\theta))$. Then $\tan(\theta) = \frac{y}{x} = \frac{-13}{-10} = \frac{13}{10}$. Since (x, y) is located in the third quadrant, where $\pi \le \theta \le \frac{3\pi}{2}$, and $\tan(\theta) = \tan(\theta + \pi)$, $\theta = \tan^{-1}(\frac{13}{10}) + \pi \approx 0.9151 + \pi \approx 4.0567$. And $r^2 = x^2 + y^2 = (-10)^2 + (-13)^2 = 269 \rightarrow r = \sqrt{269}$.

21. $x = 3 \rightarrow r\cos(\theta) = 3$ or $r = \frac{3}{\cos(\theta)} = 3\sec(\theta)$.

23. $y = 4x^2 \rightarrow r\sin(\theta) = 4\,[r\cos(\theta)]^2 = 4\,r^2\cos^2(\theta)$

Then $\sin(\theta) = 4r\cos^2(\theta)$, so $r = \frac{\sin(\theta)}{4\cos^2(\theta)} = \frac{\tan(\theta)\sec(\theta)}{4}$.

25. $x^2 + y^2 = 4y \rightarrow r^2 = 4r\sin(\theta)$. Then $r = 4\sin(\theta)$.

27. $x^2 - y^2 = x \rightarrow [r\cos(\theta)]^2 - [r\sin(\theta)]^2 = r\cos(\theta)$. Then:

$$r^2[\cos^2(\theta) - \sin^2(\theta)] = r\cos(\theta)$$
$$r[\cos^2(\theta) - \sin^2(\theta)] = \cos(\theta)$$
$$r = \frac{\cos(\theta)}{\cos^2(\theta) - \sin^2(\theta)}.$$

29. $r = 3\sin(\theta) \rightarrow r^2 = 3r\sin(\theta) \rightarrow x^2 + y^2 = 3y$.

31. $r = \frac{4}{\sin(\theta)+7\cos(\theta)} \rightarrow r\sin(\theta) + 7r\cos(\theta) = 4 \rightarrow y + 7x = 4$.

33. $r = 2\sec(\theta) = \frac{2}{\cos(\theta)} \rightarrow r\cos(\theta) = 2 \rightarrow x = 2$.

35. $r = \sqrt{r\cos(\theta) + 2} \rightarrow r^2 = r\cos(\theta) + 2 \rightarrow x^2 + y^2 = x + 2$.

37. We can choose values of θ to plug in to find points on the graph, and then see which of the given graphs contains those points.

$\theta = 0 \rightarrow r = 2 + 2\cos(0) = 4$ $\quad$ $\theta = \pi \rightarrow r = 2 + 2\cos(\pi) = 0$

$\theta = \frac{\pi}{2} \rightarrow r = 2 + 2\cos\left(\frac{\pi}{2}\right) = 2$ $\quad$ $\theta = \frac{3\pi}{2} \rightarrow r = 2 + 2\cos\left(\frac{3\pi}{2}\right) = 2$

So the matching graph should be A.

39. We can choose values of θ to plug in to find points on the graph, and then see which of the given graphs contains those points.

$\theta = 0 \rightarrow r = 4 + 3\cos(0) = 7$ $\quad$ $\theta = \pi \rightarrow r = 4 + 3\cos(\pi) = 1$

$\theta = \frac{\pi}{2} \rightarrow r = 4 + 3\cos\left(\frac{\pi}{2}\right) = 4$ $\quad$ $\theta = \frac{3\pi}{2} \rightarrow r = 4 + 3\cos\left(\frac{3\pi}{2}\right) = 4$

So the matching graph should be C.

41. $r = 5$ means that we're looking for the graph showing all points that are 5 units from the origin, so the matching graph should be E. To verify this using the Cartesian equation: $r = 5 \rightarrow r^2 = 25 \rightarrow x^2 + y^2 = 25$, which is the equation of the circle centered at the origin with radius 5.

43. We can choose values of θ to plug in to find points on the graph.

$\theta = \frac{\pi}{2} \rightarrow r = \log\left(\frac{\pi}{2}\right) \approx 0.1961$

$\theta = \pi \rightarrow r = \log(\pi) \approx 0.4971$

$\theta = \frac{3\pi}{2} \rightarrow r = \log\left(\frac{3\pi}{2}\right) \approx 0.6732$, and so on.

Observe that as θ increases its value, so does r. Therefore the matching graph should be C.

45. We can make a table of values of points satisfying the equation to see which of the given graphs contains those points.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$
r	1	$\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	-1	$\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$

So the matching graph should be D.

47. We can make a table of values of points satisfying the equation to see which of the given graphs contains those points.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	1	$1+\sqrt{2}$	-1	$1+\sqrt{2}$	1	$1-\sqrt{2}$	3	$1-\sqrt{2}$	1

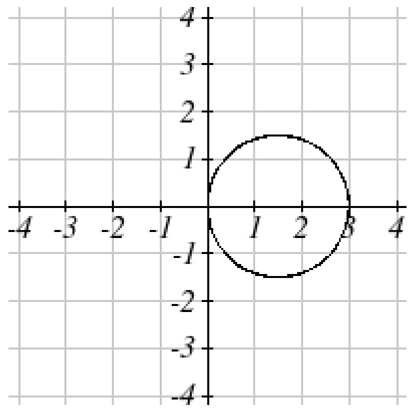
So the matching graph should be F.

49. $r = 3\cos(\theta) \rightarrow r^2 = 3r\cos(\theta) \rightarrow x^2 + y^2 = 3x$ in Cartesian coordinates. Using the method of completing square:

$x^2 - 3x + y^2 = 0$

$\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+y^2=0$, or $\left(x-\frac{3}{2}\right)^2+y^2=\frac{9}{4}$.

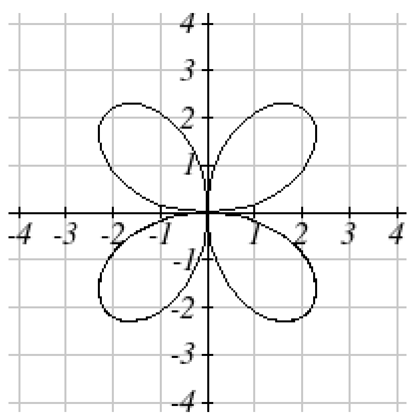
This is an equation of circle centered at $\left(\frac{3}{2},0\right)$ with radius $\frac{3}{2}$. Therefore the graph looks like:



51. We'll start with a table of values, and plot them on the graph.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	0	3	0	-3	0	3	0	-3	0

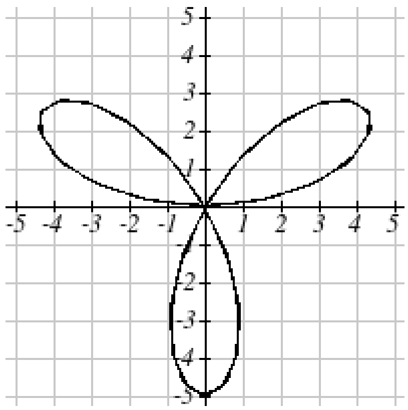
So a graph of the equation is a 4-leaf rose.



53. We'll start with a table of values, and plot them on the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
r	0	5	-5	5	0	-5	5	-5	0

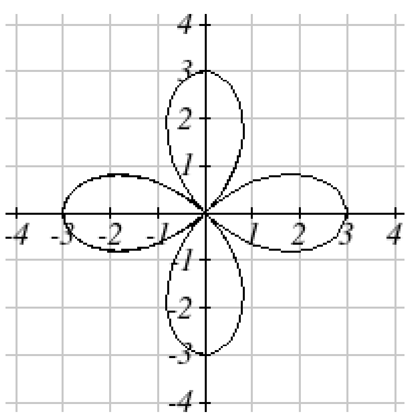
So a graph of the equation is a 3-leaf rose symmetric about the *y*- axis.



55. We'll make a table of values, and plot them on the graph.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	3	0	-3	0	3	0	-3	0	3

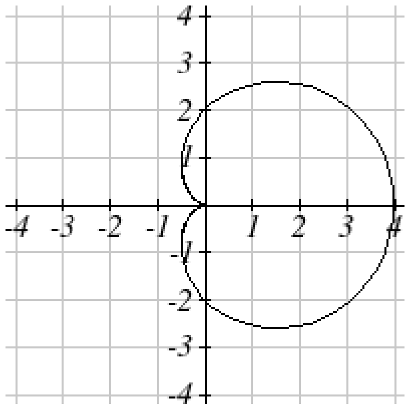
So a graph of the equation is a 4-leaf rose.



57. We'll make a table of values, and plot them on the graph.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	4	2	0	2	4

So a graph of the equation is a cardioid symmetric about the x- axis.



59. We'll start with a table of values.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	1	$\frac{2+3\sqrt{2}}{2}$	4	$\frac{2+3\sqrt{2}}{2}$	1	$\frac{2-3\sqrt{2}}{2}$	-2	$\frac{2-3\sqrt{2}}{2}$	1

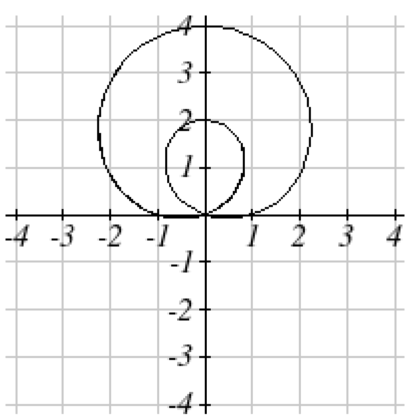
Also, when $r = 0$, $1 + 3\sin(\theta) = 0$, so:

$\sin(\theta) = -\frac{1}{3}$

$\theta \approx -0.34 + 2k\pi$

$\theta \approx \pi - (-0.34) + 2k\pi = \pi + 0.34 + 2k\pi$, where k is an integer.

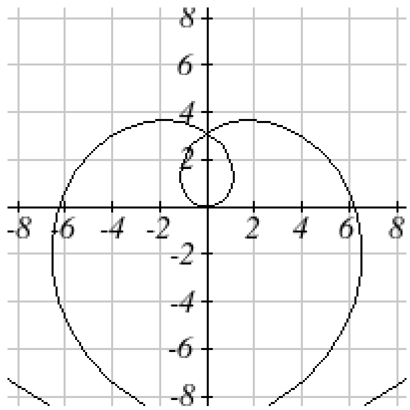
So a graph of the equation is a limaçon symmetric about the y- axis.



61. We'll start with a table of values.

θ	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
r	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Observe that as θ increases its absolute value, so does *r*. Therefore a graph of the equation should contain two spiral curves that are symmetric about the y- axis.



63. We'll start with a table of values.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	2π
r	4	$3+\sqrt{2}$	5	undef.	1	$3-\sqrt{2}$	2	$3-\sqrt{2}$	1	undef.	5	$3+\sqrt{2}$	4

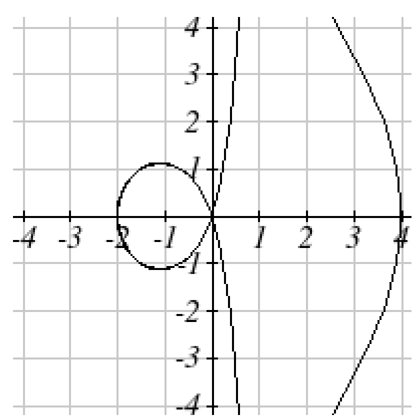
Also, when $r = 0$, $3 + \sec(\theta) = 0$, so:

$\sec(\theta) = -3$

$\theta \approx 1.9106 + 2k\pi$

$\theta \approx 2\pi - 1.9106 + 2k\pi \approx 4.3726 + 2k\pi$, where k is an integer.

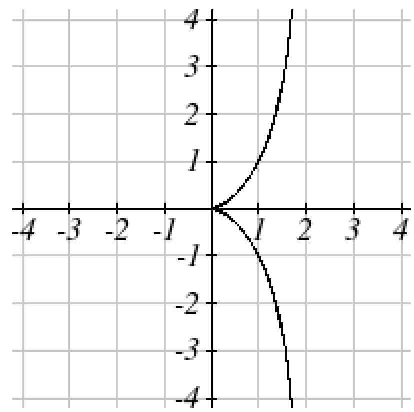
So a graph of the equation is a conchoid symmetric about the *x*-axis.



65. We'll start by choosing values of θ to plug in, to get the following table of values:

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	2π
r	0	$\sqrt{2}$	3	undef.	-3	$-\sqrt{2}$	0	$-\sqrt{2}$	-3	undef.	3	$\sqrt{2}$	0

So a graph of the equation is a cissoid symmetric about the x- axis.



8.3 Solutions to Exercises

1. $\sqrt{-9} = \sqrt{9}\sqrt{-1} = 3i$

3. $\sqrt{-6}\sqrt{-24} = \sqrt{6}\sqrt{-1}\sqrt{24}\sqrt{-1} = \sqrt{6}(2\sqrt{6})\, i^2 = 2\cdot 6\cdot i^2 = 12(-1) = -12$

5. $\frac{2+\sqrt{-12}}{2} = \frac{2+2\sqrt{3}\sqrt{-1}}{2} = 1 + i\sqrt{3}$

7. $(3 + 2i) + (5 - 3i) = 3 + 5 + 2i - 3i = 8 - i$

9. $(-5 + 3i) - (6 - i) = -5 - 6 + 3i + i = -11 + 4i$

11. $(2 + 3i)\,(4i) = 8i + 12i^2 = 8i + 12(-1) = 8i - 12$

13. $(6 - 2i)\,(5) = 30 - 10i$

15. $(2 + 3i)(4 - i) = 8 - 2i + 12i - 3i^2 = 8 + 10i - 3(-1) = 11 + 10i$

17. $(4 - 2i)(4 + 2i) = 4^2 - (2i)^2 = 16 - 4i^2 = 16 - 4(-1) = 20$

19. $\frac{3+4i}{2} = \frac{3}{2} + \frac{4i}{2} = \frac{3}{2} + 2i$

21. $\frac{-5+3i}{2i} = \frac{-5}{2i} + \frac{3i}{2i} = \frac{-5i}{2i^2} + \frac{3}{2} = \frac{-5i}{-2} + \frac{3}{2} = \frac{5i}{2} + \frac{3}{2}$

23. $\frac{2-3i}{4+3i} = \frac{(2-3i)(4-3i)}{(4+3i)(4-3i)} = \frac{8-6i-12i+9i^2}{4^2-(3i)^2} = \frac{8-18i-9}{16-9(-1)} = \frac{-18i-1}{25}$

25. $i^6 = (i^2)^3 = (-1)^3 = -1$

27. $i^{17} = (i^{16})\, i = (i^2)^8\, i = (-1)^8\, i = i$

29. $3e^{2i} = 3\cos(2) + i3\sin(2) \approx -1.248 + 2.728i$

31. $6e^{\frac{\pi}{6}i} = 6\cos\left(\frac{\pi}{6}\right) + i6\sin\left(\frac{\pi}{6}\right) = 6\left(\frac{\sqrt{3}}{2}\right) + i6\left(\frac{1}{2}\right) = 3\sqrt{3} + 3i$

33. $3e^{\frac{5\pi}{4}i} = 3\cos\left(\frac{5\pi}{4}\right) + i3\sin\left(\frac{5\pi}{4}\right) = 3\left(-\frac{\sqrt{2}}{2}\right) + i3\left(-\frac{\sqrt{2}}{2}\right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$

35. $6 = x + yi$ so $x = 6$ and $y = 0$. Also, $r^2 = x^2 + y^2 = 6^2 + 0^2 = 6^2$ so $r = 6$ (since $r \geq 0$). Also, using $x = r\cos(\theta)$: $6 = 6\cos(\theta)$, so $\theta = 0$. So the polar form is $6e^{0i}$.

37. $-4i = x + yi$ so $x = 0$ and $y = -4$. Also, $r^2 = x^2 + y^2 = 0^2 + (-4)^2 = 16$ so $r = 4$ (since $r \geq 0$). Using $y = r\sin(\theta)$: $-4 = 4\sin(\theta)$, so $\sin(\theta) = -1$, so $\theta = \frac{3\pi}{2}$. So the polar form is $4e^{\frac{3\pi}{2}i}$.

39. $2 + 2i = x + yi$ so $x = y = 2$. Then $r^2 = x^2 + y^2 = 2^2 + 2^2 = 8$, so $r = 2\sqrt{2}$ (since $r \geq 0$). Also $x = r\cos(\theta)$ and $y = r\sin(\theta)$, so:

$$2 = 2\sqrt{2}\cos(\theta) \text{ so } \cos(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \text{ and } 2 = 2\sqrt{2}\sin(\theta) \text{ so } \sin(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Therefore (x, y) is located in the quadrant I, or $\theta = \frac{\pi}{4}$. So the polar form is $2\sqrt{2}e^{\frac{\pi}{4}i}$.

41. -3 + 3i = x +yi so x = -3 and y = 3. Then $r^2 = x^2 + y^2 = (-3)^2 + 3^2 = 18$, so $r = 3\sqrt{2}$ (since $r \geq 0$). Also $x = r\cos(\theta)$ and $y = r\sin(\theta)$, so:

$-3 = 3\sqrt{2}\cos(\theta)$ so $\cos(\theta) = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ and $3 = 3\sqrt{2}\sin(\theta)$ so $\sin(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Therefore (x, y) is located in the quadrant II, or $\theta = \frac{3\pi}{4}$. So the polar form is $3\sqrt{2}e^{\frac{3\pi}{4}i}$.

43. 5 + 3i = x +yi so x = 5 and y = 3. Then $r^2 = x^2 + y^2 = 5^2 + 3^2 = 34$, so $r = \sqrt{34}$ (since $r \geq 0$). Also $x = r\cos(\theta)$ and $y = r\sin(\theta)$, so:

$5 = \sqrt{34}\cos(\theta)$ so $\cos(\theta) = \frac{5}{\sqrt{34}}$ and $3 = \sqrt{34}\sin(\theta)$ so $\sin(\theta) = \frac{5}{\sqrt{34}}$

Therefore (x, y) is located in the quadrant I, or $\theta \approx 0.54042$. So the polar form is $\sqrt{34}e^{0.54042i}$.

45. -3 + i = x +yi so x = -3 and y = 1. Then $r^2 = x^2 + y^2 = (-3)^2 + 1^2 = 10$, so $r = \sqrt{10}$ (since $r \geq 0$). Also $x = r\cos(\theta)$ and $y = r\sin(\theta)$, so:

$-3 = \sqrt{10}\cos(\theta)$ so $\cos(\theta) = \frac{-3}{\sqrt{10}}$ and $1 = \sqrt{10}\sin(\theta)$ so $\sin(\theta) = \frac{1}{\sqrt{10}}$

Therefore (x, y) is located in the quadrant II, or $\theta \approx \pi - 0.32175 \approx 2.82$. So the polar form is $\sqrt{10}e^{2.82i}$.

47. -1 – 4i = x +yi so x = -1 and y = -4. Then $r^2 = x^2 + y^2 = (-1)^2 + (-4)^2 = 17$, so $r = \sqrt{17}$ (since $r \geq 0$). Also $x = r\cos(\theta)$ and $y = r\sin(\theta)$, so:

$-1 = \sqrt{17}\cos(\theta)$ so $\cos(\theta) = \frac{-1}{\sqrt{17}}$ and $-4 = \sqrt{17}\sin(\theta)$ so $\sin(\theta) = \frac{-4}{\sqrt{17}}$

Therefore (x, y) is located in the quadrant III, or $\theta \approx \pi + 1.81577 \approx 4.9574$. So the polar form is $\sqrt{17}e^{4.9574i}$.

49. 5 – i = x +yi so x = 5 and y = -1. Then $r^2 = x^2 + y^2 = 5^2 + (-1)^2 = 26$, so $r = \sqrt{26}$ (since $r \geq 0$). Also $x = r\cos(\theta)$ and $y = r\sin(\theta)$, so:

$5 = \sqrt{26}\cos(\theta)$ so $\cos(\theta) = \frac{5}{\sqrt{26}}$ and $-1 = \sqrt{26}\sin(\theta)$ so $\sin(\theta) = \frac{-1}{\sqrt{26}}$

Therefore (x, y) is located in the quadrant IV, or $\theta \approx 2\pi - 0.1974 \approx 6.0858$. So the polar form is $\sqrt{26}e^{6.0858i}$.

51. $\left(3e^{\frac{\pi}{6}i}\right)\left(2e^{\frac{\pi}{4}i}\right) = (3)(2)\left(e^{\frac{\pi}{6}i}\right)\left(e^{\frac{\pi}{4}i}\right) = 6e^{\frac{\pi}{6}i+\frac{\pi}{4}i} = 6e^{\frac{5\pi}{12}i}$

53. $\dfrac{6e^{\frac{3\pi}{4}i}}{3e^{\frac{\pi}{6}i}} = \left(\frac{6}{3}\right)\left(\dfrac{e^{\frac{3\pi}{4}i}}{e^{\frac{\pi}{6}i}}\right) = 2e^{\frac{3\pi}{4}i-\frac{\pi}{6}i} = 2e^{\frac{7\pi}{12}i}$.

55. $\left(2e^{\frac{\pi}{4}i}\right)^{10} = (2^{10})\left(\left(e^{\frac{\pi}{4}i}\right)^{10}\right) = 1024e^{\frac{10\pi}{4}i} = 1024e^{\frac{5\pi}{2}i}$

57. $\sqrt{16e^{\frac{2\pi}{3}i}} = \sqrt{16}\sqrt{e^{\frac{2\pi}{3}i}} = 4e^{\frac{2\pi}{3}i\left(\frac{1}{2}\right)} = 4e^{\frac{\pi}{3}i}$.

59. $(2 + 2i)^8 = ((2 + 2i)^2)^4 = (4 + 8i + 4i^2)^4 = (4 + 8i - 4)^4 = (8i)^4 = 8^4i^4 = 4096$. Note that you could instead do this problem by converting $2 + 2i$ to polar form (done it problem 39) and then proceeding: $(2 + 2i)^8 = \left(2\sqrt{2}e^{\frac{\pi}{4}i}\right)^8 = \left(2\sqrt{2}\right)^8\left(e^{\frac{\pi}{4}i}\right)^8 = 4096\left(e^{\frac{\pi}{4}i\cdot 8}\right) = 4096e^{2\pi i} = 4096$.

61. $\sqrt{-3 + 3i} = (-3 + 3i)^{\frac{1}{2}}$. Let's convert $-3 + 3i$ to polar form: $-3 + 3i = x + yi$. Then x = -3 and $y = 3$. Then $r^2 = x^2 + y^2 = (-3)^2 + 3^2 = 18$ so $r = 3\sqrt{2}$ (since $r \geq 0$). Also:

$x = r\cos(\theta)$	and	$y = r\sin(\theta)$
$-3 = 3\sqrt{2}\cos(\theta)$		$3 = 3\sqrt{2}\sin(\theta)$
$\cos(\theta) = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$		$\sin(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Therefore (x, y) is located in the quadrant II, and $\theta = \frac{3\pi}{4}$. So $(-3 + 3i)^{\frac{1}{2}} = (3\sqrt{2}e^{i\frac{3\pi}{4}})^{\frac{1}{2}} = \sqrt{3\sqrt{2}}e^{i\frac{3\pi}{8}}$. To put our answer in $a + bi$ form:

$a = r\cos(\theta) = \sqrt{3\sqrt{2}}\cos(\frac{3\pi}{8}) \approx 0.78824$

$b = r\sin(\theta) = \sqrt{3\sqrt{2}}\sin(\frac{3\pi}{8}) \approx 1.903$

Thus $\sqrt{-3 + 3i} \approx 0.78824 + 1.903i$.

63. $\sqrt[3]{5 + 3i} = (5 + 3i)^{\frac{1}{3}}$. Let's convert $5 + 3i$ to polar form: $5 + 3i = x + yi$. Then $x = 5$ and $y = 3$. Then $r^2 = x^2 + y^2 = 5^2 + 3^2 = 34$ so $r = \sqrt{34}$ (since $r \geq 0$). Also:

$x = r\cos(\theta)$ and $y = r\sin(\theta)$

$5 = \sqrt{34}\cos(\theta)$ $3 = \sqrt{34}\sin(\theta)$

$\cos(\theta) = \frac{5}{\sqrt{34}}$ $\sin(\theta) = \frac{3}{\sqrt{34}}$

Therefore (x, y) is located in the quadrant I, and $\theta \approx 0.54042$. So $(5+3i)^{\frac{1}{3}} = (\sqrt{34}e^{0.54042i})^{\frac{1}{3}} = \sqrt[6]{34}\,e^{0.18014i}$. To put our answer in $a+bi$ form:

$a = r\cos(\theta) = \sqrt[6]{34}\cos(0.18014) \approx 1.771$

$b = r\sin(\theta) = \sqrt[6]{34}\sin(0.18014) \approx 0.3225$

Thus $\sqrt[3]{5+3i} \approx 1.771 + 0.3225i$.

65. If $z^5 = 2$ then $z = 2^{\frac{1}{5}}$. In the complex plane, 2 would sit on the horizontal axis at an angle of 0, giving the polar form $2e^{i0}$. Then $(2e^{i0})^{\frac{1}{5}} = 2^{\frac{1}{5}} e^0 = 2^{\frac{1}{5}}\cos(0) + i2^{\frac{1}{5}}\sin(0) = 2^{\frac{1}{5}} \approx 1.149$.

Since the angles 2π, 4π, 6π, 8π, and 10π are coterminal with the angle of 0, $2^{\frac{1}{5}}$ can be represented by turns as $(2e^{i2\pi})^{\frac{1}{5}}$, $(2e^{i4\pi})^{\frac{1}{5}}$, $(2e^{i6\pi})^{\frac{1}{5}}$, $(2e^{i8\pi})^{\frac{1}{5}}$, and $(2e^{i10\pi})^{\frac{1}{5}}$ to get all solutions.

$$(2e^{i2\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}(e^{i2\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}e^{\frac{2\pi}{5}i} = 2^{\frac{1}{5}}\cos(\tfrac{2\pi}{5}) + 2^{\frac{1}{5}}\,i\sin(\tfrac{2\pi}{5}) \approx 0.355 + 1.092i$$

$$(2e^{i4\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}(e^{i4\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}e^{\frac{4\pi}{5}i} = 2^{\frac{1}{5}}\cos(\tfrac{4\pi}{5}) + 2^{\frac{1}{5}}\,i\sin(\tfrac{4\pi}{5}) \approx -0.929 + 0.675i$$

$$(2e^{i6\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}(e^{i6\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}e^{\frac{6\pi}{5}i} = 2^{\frac{1}{5}}\cos(\tfrac{6\pi}{5}) + 2^{\frac{1}{5}}\,i\sin(\tfrac{6\pi}{5}) \approx -0.929 - 0.675i$$

$$(2e^{i8\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}(e^{i8\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}e^{\frac{8\pi}{5}i} = 2^{\frac{1}{5}}\cos(\tfrac{8\pi}{5}) + 2^{\frac{1}{5}}\,i\sin(\tfrac{8\pi}{5}) \approx 0.355 - 1.092i$$

$$(2e^{i10\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}(e^{i10\pi})^{\frac{1}{5}} = 2^{\frac{1}{5}}e^{\frac{10\pi}{5}i} == 2^{\frac{1}{5}}e^{2\pi i} = 2^{\frac{1}{5}}\cos(2\pi) + 2^{\frac{1}{5}}\,i\sin(2\pi) \approx 0.355 + 1.092i$$

Observe that for the angles $2k\pi$, where k is an integer and $k \geq 5$, the values of $(2e^{i2k\pi})^{\frac{1}{5}}$ are repeated as the same as its values when $k = 0, 1, 2, 3$, and 4. In conclusion, all complex solutions of $z^5 = 2$ are 1.149, $0.355 + 1.092i$, $-0.929 + 0.675i$, $-0.929 - 0.675i$, and $0.355 - 1.092i$.

67. If $z^6 = 1$ then $z = 1^{\frac{1}{6}}$. In the complex plane, 1 would sit on the horizontal axis at an angle of 0, giving the polar form e^{i0}. Then $(e^{i0})^{\frac{1}{6}} = e^0 = \cos(0) + i\sin(0) = 1$.

Since the angles 2π, 4π, 6π, 8π, 10π, and 12π are coterminal with the angle of 0, $1^{\frac{1}{6}}$ can be represented by turns as $(e^{i2\pi})^{\frac{1}{6}}$, $(e^{i4\pi})^{\frac{1}{6}}$, $(e^{i6\pi})^{\frac{1}{5}}$, $(e^{i8\pi})^{\frac{1}{6}}$, $(e^{i10\pi})^{\frac{1}{6}}$, and $(e^{i12\pi})^{\frac{1}{6}}$.

$$(e^{i2\pi})^{\frac{1}{6}} = e^{\frac{\pi}{3}i} = \cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$(e^{i4\pi})^{\frac{1}{6}} = e^{\frac{2\pi}{3}i} = \cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3}) = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$$

$$(e^{i6\pi})^{\frac{1}{6}} = e^{\pi i} = \cos(\pi) + i\sin(\pi) = -1$$

$$(e^{i8\pi})^{\frac{1}{6}} = e^{\frac{4\pi}{3}i} = \cos(\frac{4\pi}{3}) + i\sin(\frac{4\pi}{3}) = \frac{-1}{2} - \frac{\sqrt{3}}{2}i$$

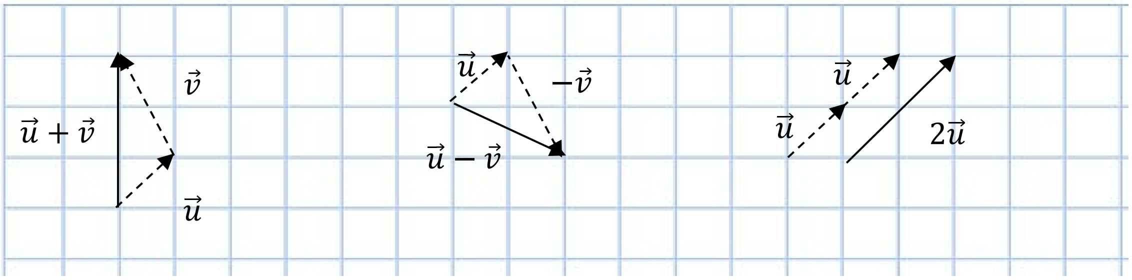
$$(e^{i10\pi})^{\frac{1}{6}} = e^{\frac{5\pi}{3}i} = \cos(\frac{5\pi}{3}) + i\sin(\frac{5\pi}{3}) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$(e^{i12\pi})^{\frac{1}{6}} = e^{2\pi i} = \cos(2\pi) + i\sin(2\pi) = 1$$

Observe that for the angles $2k\pi$, where k is an integer and $k \geq 6$, the values of $(2e^{i2k\pi})^{\frac{1}{5}}$ are repeated as the same as its values when $k = 0, 1, 2, 3, 4$, and 5. In conclusion, all complex solutions of $z^6 = 1$ are 1, $\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $\frac{-1}{2} + \frac{\sqrt{3}}{2}i$, -1, $\frac{-1}{2} - \frac{\sqrt{3}}{2}i$, and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$.

8.4 Solutions to Exercises

1. Initial point (4,0); terminal point (0,2). The vector component form is $< x_1 - x_2, y_1 - y_2 >$.
$< 0 - 4, 2 - 0 > = < -4, 2 >$

3. 

5. $\vec{u} =< 1,1 >$ and $\vec{v} =< -1,2 >$. The vector we need is $< -4,5 >$. To get these components as a combination of $\vec{u}$ and $\vec{v}$, we need to find a and b such that $a \cdot 1 + b \cdot (-1) = -4$ and $a \cdot 1 + b \cdot 2 = 5$. Solving this system gives $a = -1$ and $b = 3$, so the vector is $3\vec{v} - \vec{u}$.

7. The component form is $< 6\cos 45°,\ 6\sin 45° > = < 3\sqrt{2}, 3\sqrt{2} >$.

9. The component form is $< 8\cos 220°, 8\sin 220° > \approx < -6.128, -5.142 >$.

11. Magnitude: $|\boldsymbol{v}| = \sqrt{0^2 + 4^2} = 4$; direction: $\tan\theta = \frac{4}{0}$ so $\theta = 90°$

13. Magnitude: $|\boldsymbol{v}| = \sqrt{6^2 + 5^2} = \sqrt{61} = 7.81$, direction: $\tan\theta = \frac{5}{6}, \theta = \tan^{-1}\frac{5}{6} \approx 39.806°$ (first quadrant).

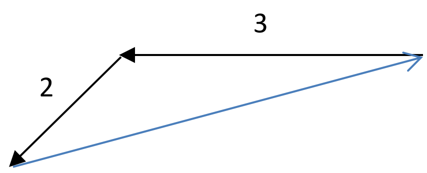
15. Magnitude: $|\boldsymbol{v}| = \sqrt{(-2)^2 + 1^2} = \sqrt{5} \approx 2.236$; direction: $\tan\theta = \frac{1}{-2} = -26.565$ which is $180° - 26.565° = 153.435°$(second quadrant).

17. Magnitude: $|\boldsymbol{v}| = \sqrt{2^2 + (-5)^2} = \sqrt{29} \approx 5.385$, direction: $\tan\theta = \frac{-5}{2}, \theta = \tan^{-1}\frac{-5}{2}, \approx -68.199$ which is $360° - 68.199° = 291.801°$(fourth quadrant).

19. Magnitude: $|\boldsymbol{v}| = \sqrt{(-4)^2 + (-6)^2} = \sqrt{52} \approx 7.211$, direction: $\tan\theta = \frac{-6}{-4}, \theta = \tan^{-1}\frac{-6}{-4}, \approx 56.3°$ which is $180° + 56.3° = 236.3°$(third quadrant).

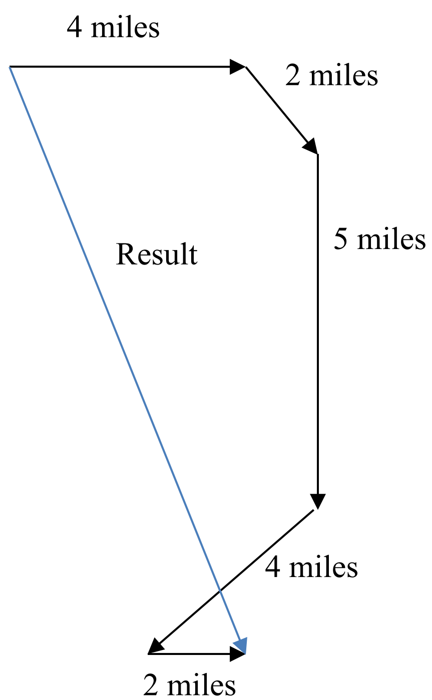
21. $\vec{u} + \vec{v} =< 2 + 1, -3 + 5 > = < 3,2 >$; $\vec{u} - \vec{v} =< 2 - 1, -3 - 5 >= < 1, -8 >$; $2\vec{u} = 2 < 2, -3 >$ and $3\vec{v} = 3 < 1, 5 >$ so $2\vec{u} - 3\vec{v} =< 4 - 3, -6 - 15 >=< 1, -21 >$.

23.



The first part of her walk can be defined as a vector form of $< -3,0 >$. The second part of her walk can be defined as $< 2\cos(225°), 2\sin(225°) >$. Then the total is $< -3 + 2\cos 225°, 0 + 2\sin 225° > = < -3 - \sqrt{2}, -\sqrt{2} >$. The magnitude is $\sqrt{\left(-3 - \sqrt{2}\right)^2 + \left(-\sqrt{2}\right)^2} = \sqrt{21.485} \approx 4.635$ miles. Direction: $\tan\theta = \frac{\sqrt{2}}{3+\sqrt{2}} = 0.3203$, $\theta \approx 17.76$ north of east.

25.



How far they have walked: $4 + 2 + 5 + 4 + 2 = 17$ miles. How far they had to walk home, if they had walked straight home: the 5 parts of their walk can be considered into 5 vector forms:

$< 4, 0 >, < 2\cos 45°, \ -2\sin 45° >, < 0, -5 >, < -4\cos 45°, -4\sin 45° >$, and $< 2, 0 >$.

Total horizontal components of the vectors:

$4 + 2\cos 45° + 0 - 4\cos 45° + 2 = \ 4 + \sqrt{2} + 0 - 2\sqrt{2} + 2 = 6 - \sqrt{2}$

Total vertical components of the vectors:

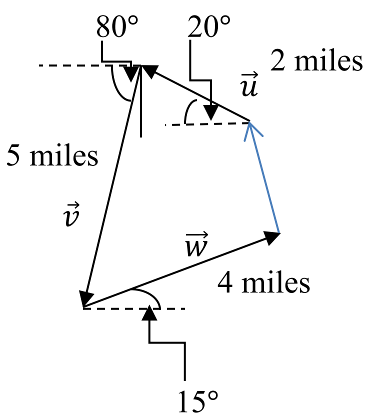
$0 - 2\sin 45° - 5 - 4\sin 45° + 0 = -\sqrt{2} - 5 - 2\sqrt{2} >= -5 - 3\sqrt{2}$

The total distance is the magnitude of this vector: $\sqrt{(6-\sqrt{2})^2 + (-5-3\sqrt{2})^2} =$

$\sqrt{21.029 + 85.426} \approx 10.318$ miles.

27. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = < -8 + 0 + 4, -5 + 1 - 7 > = < -4, -11 >$

29.



$$\vec{u} = < 3\cos 160°, 3\sin 160° >$$

$$\vec{v} = < 5\cos 260°, 5\sin 260° >$$

$$\vec{w} = < 4\cos 15°, 4\sin 15° >$$

$\vec{u} + \vec{v} + \vec{w} = < 3\cos 160° + 5\cos 260° + 4\cos 15°, 3\sin 160° + 5\sin 260° + 4\sin 15° >$

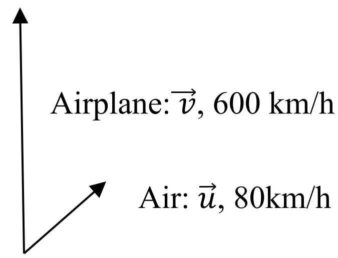
$= < 0.1764, -2.8627 >$

Note that this vector represents the person's displacement from home, so the path to return home is the opposite of this vector, or $< -0.1764, 2.8627 >$. To find its magnitude:

$$|\vec{u} + \vec{v} + \vec{w}| \ = < -0.1764, 2.8627 > = \sqrt{0.03111 + 8.1950} = 2.868 \text{ miles.}$$

Directions: $\tan\theta = \frac{2.8627}{-0.1764}$, so $\theta = 93.526°$, which is 86.474° North of West or 90 – 86.474 = 3.526° West of North.

31.

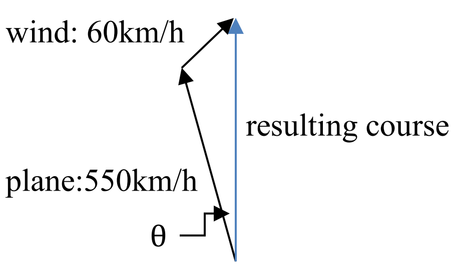


airplane speed: $\vec{v} = < 0, 600 >$

air speed: $\vec{u} = < 80\cos 45°, 80\sin 45° >$

Effective airplane speed in relation to the ground $\vec{w} = \vec{u} + \vec{v} < 0 + 80\cos 45°, 600 + 80\sin 45° > = < 40\sqrt{2}, 600 + 40\sqrt{2} >$. Then $|\vec{w}| = \sqrt{(40\sqrt{2})^2 + (600 + 40\sqrt{2})^2} = 659\text{km/h}$. To find the direction to the horizontal axis: $\tan\theta = \frac{600+40\sqrt{2}}{40\sqrt{2}} = 11.607$. Then $\theta = 85.076°$. So the plane will fly $(90 - 85.076)° = 4.924°$ off the course.

33. Suppose the plane flies at an angle θ° to north of west axis Then its vector components are $< 550\cos\theta, 550\sin\theta >$. The vector components for wind are $< 60\cos 45°, 60\sin 45° >$.



Since the plane needs to head due north, the horizontal components of the vectors add to zero:

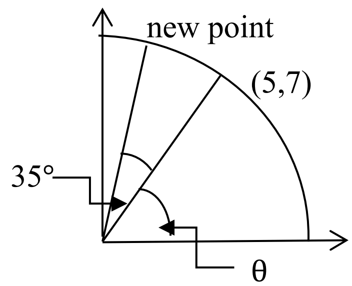
$550\cos(90° + \theta) + 60\cos 45° = 0$

$550\cos(90° + \theta) = -60\cos 45 = 30\sqrt{2}$

$\cos(90° + \theta) = \frac{30\sqrt{2}}{550}$

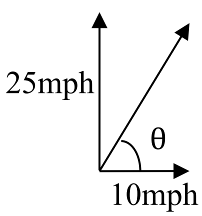
$90° + \theta$ =85.576° or 94. 424°. Since $90° + \theta$ should give an obtuse angle, we use the latter solution, and we conclude that the plane should fly 4.424° degrees west of north.

35.



Suppose the angle the point (5,7) makes with the horizontal axis is θ, then $\tan\theta = \frac{7}{5}$, $\theta = \tan^{-1}\frac{7}{5} = 54.46°$. The radius of the quarter circle $= \sqrt{5^2 + 7^2} = \sqrt{74} = 8.602$. The angle which formed by the rotation from the point (5, 7) is =35°, so the new angle formed by the rotation from the horizontal axis $= 54.46° + 35° = 89.46°$. So the new coordinate points are: $(8.602\cos 89.46°, 8.602\sin 89.46°) = (0.081, 8.602)$.

37.



$\tan\theta = \frac{25}{10}$, therefore $\theta = 68.128°$; in relation to car's forward direction it is $= 90 - 68.128 = 21.80°$.

Section 8.5 solutions

1. $6 \cdot 10 \cdot \cos(75°) = 15.529$ 3. $(0)(-3) + (4)(0) = 0$ 5. $(-2)(-10) + (1)(13) = 33$

7. $\cos^{-1}\left(\frac{0}{\sqrt{4}\sqrt{3}}\right) = 90°$ 9. $\cos^{-1}\left(\frac{(2)(1)+(4)(-3)}{\sqrt{2^2+4^2}\sqrt{1^2+(-3)^2}}\right) = 135°$

11. $\cos^{-1}\left(\frac{(4)(8)+(2)(4)}{\sqrt{4^2+8^2}\sqrt{2^2+4^2}}\right) = 0°$ 13. $(2)(k) + (7)(4) = 0$, $k = -14$

15. $\frac{(8)(1)+(-4)(-3)}{\sqrt{1^2+(-3)^2}} = 6.325$

17. $\left(\frac{(-6)(1)+(10)(-3)}{\sqrt{1^2+(-3)^2}^2}\right)\langle 1,-3\rangle = \langle -3.6, 10.8\rangle$

19. The vectors are $\langle 2,3\rangle$ and $\langle -5,-2\rangle$. The acute angle between the vectors is 34.509°

21. 14.142 pounds

23. $\langle 10\cos(10°), 10\sin(10°)\rangle \cdot \langle 0,-20\rangle$, so 34.7296 ft-lbs

25. $40 \cdot 120 \cdot \cos(25°) = 4350.277$ft-lbs

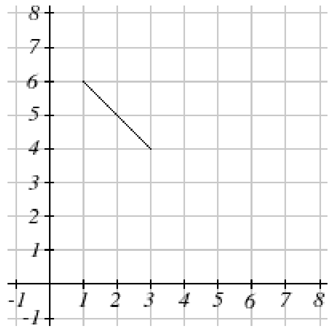
8.6 Solutions to Exercises

1. The first equation $\mathrm{x} = \mathrm{t}$ can be substituted into the second equation to get $\mathrm{y}(x) = \mathrm{x}^2 - 1$, corresponding to graph C. [Note: earlier versions of the textbook contained an error in which the graph was not shown.]

3. Given $x(t) = 4\sin(t)$, $y(t) = 2\cos(t)$: $\frac{x}{4} = \sin(t), \frac{y}{2} = \cos(t)$. We know $\sin^2(t) + \cos^2(t) = 1$, so $\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1$. This is the form of an ellipse containing the points $(\pm 4, 0)$ and $(0, \pm 2)$, so the graph is E.

5. From the first equation, $t = x - 2$. Substituting into the second equation, $y = 3 - 2(x-2) = 3 - 2x + 4 = -2x + 7$, a linear equation through (0, 7) with slope -2, so it corresponds to graph F.

7. It appears that $x(t)$ and $y(t)$ are both sinusoidal functions: $x(t) = \sin(t) + 2$ and $y(t) = -\sin(t) + 5$. Using the substitution $\sin(t) = x - 2$ from the first equation into the second equation, we get $y = -(x-2) + 5 = -x + 7$, a line with slope -1 and *y*-intercept 7. Note that since $-1 \le \sin(t) \le 1$, *x* can only range from 1 to 3, and *y* ranges from 4 to 6, giving us just a portion of the line.

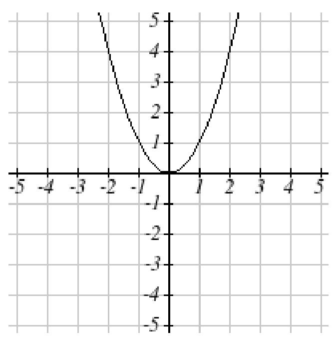


9. We can identify (x, y) pairs on the graph, as follows:

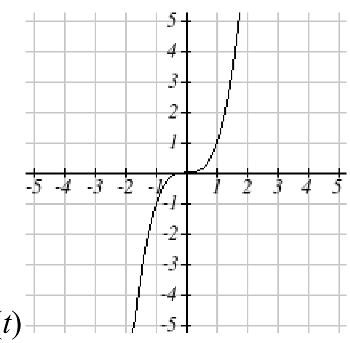
x	y
0	0
1	-1,1
2	$\approx -3{,}3$
3	$\approx -5{,}5$

It appears that this relationship could be described by the equation $y = \pm\sqrt{x^3}$, so we can use the parametric equations$x(t) = t^2$ and $y(t) = t^3$.

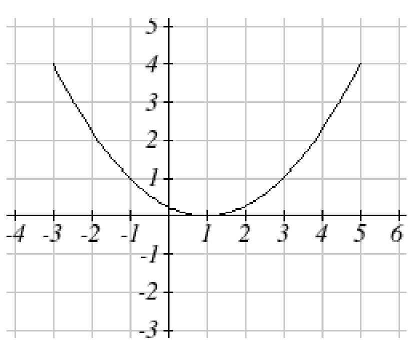
$x(t)$



$y(t)$



11. From the first equation, we get $t = \frac{1}{2}(x-1)$, and since *t* ranges from -2 to 2, *x* must range from -3 to 5. Substituting into the second equation, we get $y = \left(\frac{1}{2}(x-1)\right)^2 = \frac{1}{4}(x-1)^2$.



13. From the first equation, $t = 5 - x$. Substituting into the second equation, $y = 8 - 2(5 - x) = 8 - 10 + 2x$, so the Cartesian equation is $y = 2x - 2$.

15. From the first equation, $t = \frac{x-1}{2}$. Substituting into the second equation, $y = 3\sqrt{\left(\frac{x-1}{2}\right)}$.

17. From the first equation, $t = \ln\left(\frac{x}{2}\right)$. Substituting into the second equation, $y = 1 - 5\ln\left(\frac{x}{2}\right)$.

19. From the second equation, $t = \frac{y}{2}$. Substituting into the first equation, $x = \left(\frac{y}{2}\right)^3 - \frac{y}{2}$.

21. Note that the second equation can be written as $y = (e^{2t})^3$. Then substituting $x = e^{2t}$ from the first equation into this new equation, we get $y = x^3$.

23. From the first equation, $\cos(t) = \frac{x}{4}$. From the second equation, $\sin(t) = \frac{y}{5}$. Since $\sin^2(t) + \cos^2(t) = 1$, we get $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$.

25. The simplest solution is to let $x(t) = t$. Then substituting *t* for *x* into the given equation, we get $y(t) = 3t^2$ +3.

27. Since the given equation is solved for *x*, the simplest solution is to let $y(t) = t$. Then substituting *t* for *y* into the given equation, we get $(t) = 3\log(t) + t$.

29. Note that this is an equation for an ellipse passing through points $(\pm 2, 0)$ and $(0, \pm 3)$. We can think of this as the unit circle $(\cos(t), \sin(t))$ stretched 2 units horizontally and 3 units vertically, so $x(t) = 2\cos(t)$ and $y(t) = 3\sin(t)$.

31. There are several possible answers, two of which are included here. It appears that the given graph is the graph of $y = \sqrt[3]{x} + 2$, so one possible solution is $x(t) = t$ and $y(t) = \sqrt[3]{t} + 2$. We could also look at the equation as $x = (y - 2)^3$. To parameterize this, we can let $x(t) = t^3$. We'd then need $t = y - 2$, so $y(t) = t + 2$.

33.

33. The given graph appears to be the graph of $y = -(x+1)^2$. One possible solution is to let $y(t) = -t^2$. We'd then need $t = x + 1$, so $x(t) = t - 1$.

35. Since the Cartesian graph is a line, we can allow both $x(t)$ and $y(t)$ to be linear, i.e. $x(t) = m_1 t + b_1$ and $y(t) = m_2 t + b_2$. When considering x in terms of t, the slope of the line will be $m_1 = \frac{(2-(-1))}{1} = 3$. Note that b_1 is the value of x when $t = 0$, so $b_1 = -1$. Then $x(t) = 3t - 1$. Likewise, $y(t)$ will have a slope of $m_2 = \frac{5-3}{0-1} = -2$, and $b_2 = y(0) = 5$. Then $y(t) = -2t + 5$.

37. Since the range of the cosine function is [-1, 1] and the range of *x* shown is [-4, 4], we can conclude $a = 4$. By an analogous argument, $c = 6$. then $x(t) = 4\cos(bt)$ and $y(t) = 6\sin(dt)$

Since $x(0) = 4\cos(b \cdot 0) = 4$ for any value of b, and $y(0) = 6\sin(d \cdot 0) = 0$ for any value of d, the point $(4, 0)$ is where $t = 0$. If we trace along the graph until we return to this point, the *x*-coordinate moves from its maximum value of 4 to its minimum of value -4 and back exactly 3 times, while the *y*-coordinate only reaches its maximum and minimum value, 6 and -6 respectively, exactly once. This means that the period of $y(t)$ must be 3 times as large as the period of $x(t)$. It doesn't matter what the periods actually are, as long as this ratio is preserved. Recall that *b* and *d* have an inverse relation to the period ($b = \frac{2\pi}{\text{period of } x}$, and similarly for *d* and *y*), so *d* must be one third of *b* for *y* to have three times the period of *x*. So let's let $b = 3$ and $d = 1$. Then $x(t) = 4\cos(3t)$ and $y(t) = 6\sin(t)$.

39. Since the range of the cosine function is [-1, 1] and the range of *x* shown is [-4, 4], we can conclude $a = 4$. By an analogous argument, $c = 3$. then $x(t) = 4\cos(bt)$ and $y(t) = 3\sin(dt)$.

Since $x(0) = 4\cos(b \cdot 0) = 4$ for any value of b, and $y(0) = 3\sin(d \cdot 0) = 0$ for any value of d, the point $(4, 0)$ is where $t = 0$. From this point, imagine tracing the figure until the whole figure is drawn and we return to this starting point. (Note that in order to do that, when reaching (-4, 3) or (-4, -3), we must backtrack along the same path.) The *x*-coordinate moves from its maximum value of 4 to its minimum of value -4 and back twice, while the *y*-coordinate moves through its maximum and minimum values, 3 and -3 respectively, three times. If we think about compressing the graphs of the standard sine and cosine graphs to increase the period accordingly (as in Chapter 6), we need $b = 2$ (to change the cosine period from 2π to π) and $d = 3$ (to change the sine period from 2π to $\frac{2\pi}{3}$). Then $x(t) = 4\cos(2t)$ and $y(t) = 3\sin(3t)$.

41. Since distance = rate · time, we can model the horizontal distance at $x(t) = 15t$. Then $t = \frac{x}{15}$. Substituting this into the $y(t)$ equation, we get $y(x) = -16\left(\frac{x}{15}\right)^2 + 20\left(\frac{x}{15}\right)$.

43. We'll model the motion around the larger circle, $x_L(t)$ and $y_L(t)$, and around the smaller circle relative to the position on the larger circle, $x_S(t)$ and $y_S(t)$, and add the *x* and *y* components from each to get our final answer.

Since the larger circle has diameter 40, its radius is 20. The motion starts in the center with regard to its horizontal position, at its lowest vertical point, so if we model x_L with a sine function and y_L with a cosine function, we will not have to find a phase shift for either. If we impose a coordinate system with the origin on the ground directly below the center of the circle, we get $x_L(t) = 20\sin(Bt)$ and $y_L(t) = -20\cos(Bt) + 35$. The period of the large arm is 5 seconds, so $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{5}$. Then $x_L(t) = 20\sin\left(\frac{2\pi}{5}t\right)$ and $y_L(t) = -20\cos\left(\frac{2\pi}{5}t\right) + 35$.

The small arm has radius 8 and period 2, and also starts at its lowest point, so by similar arguments, $x_S(t) = 8\sin(\pi t)$ and $y_S(t) = -8\cos(\pi t)$.

Adding the coordinates together, we get $x(t) = 20\sin\left(\frac{2\pi}{5}t\right) + 8\sin(\pi t)$ and $y(t) = -20\cos\left(\frac{2\pi}{5}t\right) - 8\cos(\pi t) + 35$.