3.1 Solutions to Exercises

1. (a) $f(x)$ will approach $+\infty$ as x approaches ∞.
   (b) $f(x)$ will still approach $+\infty$ as x approaches $-\infty$, because any negative integer x will become positive if it is raised to an even exponent, in this case, x^4

3. (a) $f(x)$ will approach $+\infty$ as x approaches ∞.
   (b) $f(x)$ will approach $-\infty$ as x approaches $-\infty$, because x is raised to an odd power, in this case, x^3.

5. (a) $f(x)$ will approach $-\infty$ as x approaches ∞, because every number is multiplied by -1.
   (b) $f(x)$ will approach $-\infty$ as x approaches $-\infty$, since any negative number raised to an even power (in this case 2) is positive, but when it's multiplied by -1, it becomes negative.

7. (a) $f(x)$ will approach $-\infty$ as x approaches ∞, because any positive number raised to any power will remain positive, but when it's multiplied by -1, it becomes negative.
   (b) $f(x)$ will approach ∞ as x approaches $-\infty$, because any negative number raised to an odd power will remain negative, but when it's multiplied by -1, it becomes positive.

9. (a) The degree is 7.
   (b) The leading coefficient is 4.

11. (a) The degree is 2.
    (b) The leading coefficient is -1.

13. (a) The degree is 4.
    (b) The leading coefficient is -2.

15. (a) $(2x + 3)(x - 4)(3x + 1) = (2x^2 - 5x - 12)(3x + 1) = 6x^3 - 13x^2 - 41x - 12$
    (b) The leading coefficient is 6.
    (c) The degree is 3.

17. (a) The leading coefficient is negative, so as $x \to +\infty$ the function will approach $-\infty$.
    (b) The leading coefficient is negative, and the polynomial has even degree so as $x \to -\infty$ the function will approach $-\infty$.

19. (a) The leading coefficient is positive, so as $x \to +\infty$, the function will approach $+\infty$.
    (b) The leading coefficient is positive, and the polynomial has even degree so as $x \to -\infty$, the function will approach $+\infty$.

21. (a) Every polynomial of degree n has a maximum of n x-intercepts. In this case $n = 5$ so we get a maximum of five x-intercepts.
    (b) The number of turning points of a polynomial of degree n is $n - 1$. In this case $n = 5$ so we get four turning points.

23. Knowing that an nth degree polynomial can have a maximum of $n - 1$ turning points we get that this function with two turning points could have a minimum possible degree of three.

25. Knowing that an nth degree polynomial can have a maximum of $n - 1$ turning points we get that this function with four turning points could have a minimum possible degree of five.

27. Knowing that an nth degree polynomial can have a maximum of $n - 1$ turning points we get that this function with two turning points could have a minimum possible degree of three.

29. Knowing that an nth degree polynomial can have a maximum of $n - 1$ turning points we get that this function with four turning points could have a minimum possible degree of five.

31. (a) To get our vertical intercept of our function we plug in zero for t we get $f(0) = 2((0) - 1)((0) + 2)((0) - 3) = 12$. Therefore our vertical intercept is $(0,12)$
    (b) To get our horizontal intercepts when our function is a series of products we look for when we can any of the products equal to zero. For $f(t)$ we get $t = -2, 1, 3$. Therefore our horizontal intercepts are $(-2,0)$, $(1,0)$ and $(3,0)$.

33. (a) To get our vertical intercept of our function we plug in zero for n we get $g(0) = -2((3(0) - 1)(2(0) + 1) = 2$. Therefore our vertical intercept is $(0,2)$
    (b) To get our horizontal intercepts when our function is a series of products we look for when we can any of the products equal to zero. For $g(n)$ we get $n = \frac{1}{3}, \frac{-1}{2}$. Therefore our horizontal intercepts are $(\frac{1}{3}, 0)$ and $(\frac{-1}{2}, 0)$.

3.2 Solutions to Exercises

1. $f(x) = x^2 - 4x + 1$ 　　　　　　　　　　 3. $f(x) = -2x^2 + 8x - 1$

5. $f(x) = \frac{1}{2}x^2 - 3x + \frac{7}{2}$

7. Vertex: $\left(-\frac{10}{4}, -\frac{1}{2}\right)$ x-intercepts: $(-3,0)(-2,0)$ y-intercept: $(0,12)$

9. Vertex: $\left(\frac{10}{4}, -\frac{29}{2}\right)$ x-intercepts: $(5,0)(-1,0)$ y-intercept: $(0,4)$

11. Vertex: $\left(\frac{3}{4}, 1.25\right)$ x-intercepts: $\pm\sqrt{5}$ y-intercept: $(0,-1)$

13. $f(x) = (x - 6)^2 - 4$ 　　　　　　　　 15. $h(x) = 2(x + 2)^2 - 18$

17. We have a known $a, h,$ and k. We are trying to find b and c, to put the equation into quadratic form. Since we have the vertex, $(2, -7)$ and $a = -8$, we can put the equation into vertex form, $f(x) = -8(x - 2)^2 - 7$ and then change that into quadratic form. To do this, we start by foiling $(x - 2)^2$ and algebraically continuing until we have the form $f(x) = ax^2 + bx + c$. We get $f(x) = -8x^2 + 32x - 39$, so $b = 32$ and $c = $ -39.

19. $f(x) = -\frac{2}{3}x^2 - \frac{4}{3}x + 2$

21. $f(x) = \frac{3}{5}x^2 - \frac{21}{5x} + 6$

23. $-\frac{b}{2a}$ is the x-coordinate of the vertex, and we are given the x-coordinate of the vertex to be 4, we can set $-\frac{b}{2a}$ equal to 4, and solve for b, which gives $b = -8a$. The b in the vertex formula is the same as the b in the general form of a quadratic equation $y = ax^2 + bx + c$, so we can substitute $-8a$ for b and -4 for c (the y-intercept) into the quadratic equation: $y = ax^2 - 8ax - 4$. Plugging in the x and y coordinates from the y intercept gives $0 = 16a - 32a - 4$, and solving for a gives $a = \frac{-1}{4}$. After plugging a back in we get $y = -\frac{1}{4}x^2 - 8\left(-\frac{1}{4}\right)x - 4$ which simplifies to $y = -\frac{1}{4}x^2 + 2x - 4$.

25. $-\frac{b}{2a}$ is the x-coordinate of the vertex, and we are given the x-coordinate of the vertex to be -3, we can set $-\frac{b}{2a}$ equal to -3, and solve for b, which gives $b = 6a$. So our equation is $y = ax^2 + 6ax + c$. Plugging in the vertex coordinates for x and y allow us to solve for c, which gives $c = 2 + 9a.$ Plugging c back into y gives $y = ax^2 + 6ax + 2 + 9a$. To solve for a we plug in values from the other point, $-2 = 9a + 18a + 2 + 9a$ which gives $a = -\frac{1}{9}$. We have b and c in terms of a, so we can find them easily now that we know the value of a. So, the final equation is $y = -\frac{1}{9}x^2 - \frac{2}{3}x + 1.$

27. For this problem, part (a) asks for the height when $t = 0$, so solving for $h(0)$ will give us our launching height. In part (b), we are trying to find the peak of the trajectory, which is the same as the vertex, so solving for k will give us the maximum height. In part (c), we are asked to solve

for t when $h(t) = 0$. We can do this by using the quadratic formula to solve for t.

(a) 234 m

(b) 2909.56 m

(c) 47.735 sec

29. See the explanation for problem 27 for hints on how to do this problem.

(a) 3 ft

(b) 111 ft

(c) 72.48 ft

Problem 31

31. The volume of the box can be expressed as $V = 6 * x *$ x or $V = 6x^2$. So if we want the volume to be 1000, we end up with the expression $6x^2 - 1000 = 0$. Solving this equation for x using the quadratic formula we get $= \pm 10\sqrt{\frac{5}{3}}$.
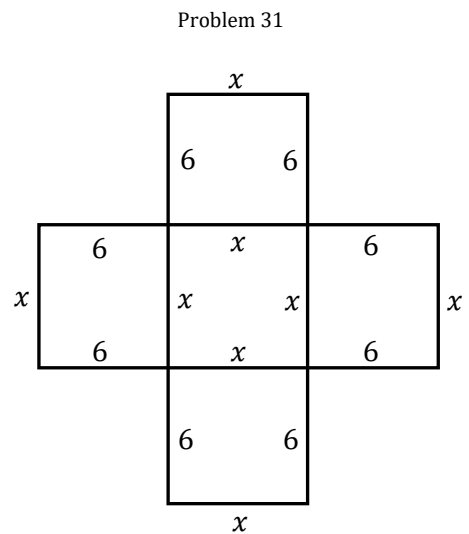
Because we cannot have negative length, we are left with $x = 10\sqrt{\frac{5}{3}} \approx 12.90$. So the length of the side of our box is $12 +$ $12.90 = 24.90$. So, our piece of cardboard is $24.90 *$ $24.90 = 620$.

33. Picking x to be our vertical side, we are left with $500 - 3x$ for the remaining two sides. Because there are two of them, we divide that by two, so that each side is $\frac{500-3x}{2} = 250 - \frac{3x}{2}$. So, the area can be expressed as $=$ $250x - \frac{3}{2}x^2$. When this function has a maximum, the area of the enclosure will be maximized. This a concave down parabola and thus has a maximum point at its vertex. The x-coordinate of the vertex is $-\frac{b}{2a}$, which we can calculate to be $x = \frac{-250}{-2\left(\frac{3}{2}\right)} = 83\frac{1}{3}$. This is the

Problem 33

dimension of the enclosure x. To find the long dimension, we can plug this value for x into our
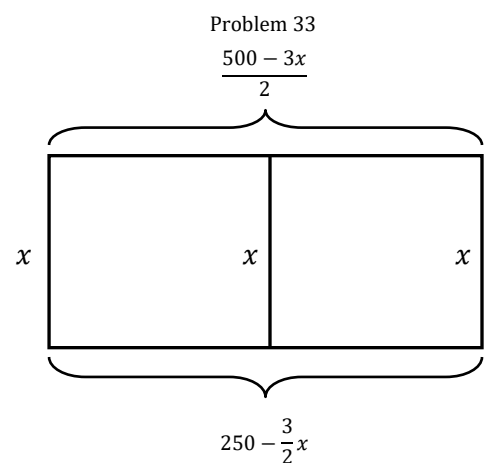
expression $250 - \frac{3}{2}x$ which $= 125$. So, the dimensions are $83\frac{1}{3}$ ft for three vertical sides and 125 ft for the two long sides.

35. Let x represent the length in cm of the piece of wire that is bent into the shape of a circle. Then the length of wire left to be bent into the shape of a square is $56 - x$ cm. The length x of wire will wrap around the circle, forming the circumference, so $x = 2\pi r$ where r is the radius of the circle. Thus $r = \frac{x}{2\pi}$, and the area of the circle is $A_{cir} = \pi r^2 = \pi \left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$. Since the remaining length of $56 - x$ is bent into a square, then each of the four sides will have length $\frac{56-x}{4}$.

Thus, the area of the square is $A_{sq} = \left(\frac{56-x}{4}\right)^2 = \frac{(56-x)^2}{16}$. The total area for both figures is $A =$

$\frac{x^2}{4\pi} + \frac{(56-x)^2}{16} = \left(\frac{1}{4\pi} + \frac{1}{16}\right)x^2 - 7x + 196$. The graph of this equation is a parabola that opens upward. The minimum value of A will occur at the vertex. Using the vertex formula,

$$x = -\frac{-7}{2\left(\frac{1}{4\pi}+\frac{1}{16}\right)} = \frac{7}{\frac{1}{2\pi}+\frac{1}{8}} \cdot \frac{8\pi}{8\pi} = \frac{56\pi}{4+\pi} \approx 24.6344 \text{ cm. Thus, the circumference of the circle}$$

when the total area A is minimum is $\dfrac{56\pi}{4+\pi}$ cm, or approximately 24.6344 cm.

37. Let x represent the price, in dollars, of each ticket. Let y represent the number of spectators attending each game. The slope of a line relating these quantities is $\frac{\Delta y}{\Delta x} = \frac{26,000-31,000}{11-9} = -2500$ people per dollar. The equation of the line can be expressed in point-slope form as $y -$ $26,000 = -2500(x - 11) \Rightarrow y = -2500x + 53,500$. The revenue R, in dollars, for each game is the product (ticket price)(the number of spectators in attendance). We have $R = x(-2500x + 53,500) = -2500x^2 + 53,500x$. The graph of this equation is a parabola that opens downward. Its maximum value occurs at the vertex. Using the vertex formula, $x = -\frac{53,500}{2(-2500)} = 10.7$. Therefore, a ticket price of $10.70 would maximize revenue.

39. (a) To get the equation of the mountain side, we know that for every twenty feet in the x direction we get a rise of two feet. Then our rise over run (slope) will be $\frac{2 ft}{20 ft} = \frac{1}{10}$. Because our graph of the mountain side starts at $(0,0)$ we know our vertical intercept is zero. Then to get the

equation for the height of the balloon $f(x)$ above the mountain side $mt(x)$ we get $f(x) -$

$mt(x) = \frac{-1}{1250}x^2 + 45x - \frac{1}{10}x = \frac{-1}{1250}x^2 + \frac{449}{10}x.$ $f(x) - mt(x)$ is a concave down parabola and

therefore has a maximum value at its vertex at $x = \frac{-b}{2a} = \frac{-449}{10\left(-\frac{2}{1250}\right)} = 28062.5$. Then plugging this

value into $f(x) - mt(x)$ we get $f(28062.5) - mt(28062.5) = \frac{-1}{1250}(28062.5)^2 +$

$\frac{449}{10}(28062.5) = 632809.375$ ft.

(b) Given $f(x) = \frac{-1}{1250}x^2 + 45x$ is the balloon's height above ground level then there is a

maximum point at the vertex of the parabola with an $x = \frac{-b}{2a} = -\frac{45}{\frac{-2}{1250}} = 28125$. Plugging this

value back into $f(x)$ we get 632812.5 feet as our maximum height above ground level.

(c) To find where the balloon lands we solve for the zeros of $f(x) - mt(x) = \frac{-1}{1250}x^2 +$

$\frac{449}{10}x$. Then we get $\frac{-1}{1250}x^2 + \frac{449}{10}x = 0$ therefore $\frac{-1}{1250}x = \frac{-449}{10}$ therefore $x = 56125$ feet.

(d) To find when the balloon is 50ft off the ground we set $f(x) = 50$ and solve for x. We

can write this in the expression $\frac{-1}{1250}x^2 + \frac{449}{10}x = 50$ or $\frac{-1}{1250}x^2 + \frac{449}{10}x - 50 = 0$. Then using the

quadratic equation we get $x = 1.11$ and 56248.9ft. Although it would appear that we have two

values for when the balloon is 50 ft high looking at pt (c) we can see that the balloon will have

already landed before it reaches 56248.9ft so our only valid result is at 1.11ft.

3.3 Solutions to Exercises

1 - 5 To find the C intercept, evaluate $c(t)$. To find the t-intercept, solve $C(t) = 0$.

1. (a) C intercept at (0, 48)

    (b) t intercepts at (4,0), (-1,0), (6,0)

3. (a) C intercept at (0,0)

    (b) t intercepts at (2,0), (-1,0), (0,0)

5. $C(t) = 2t^4 - 8t^3 + 6t^2 = 2t^2(t^2 - 4t + 3) = 2t^2(t - 1)(t - 3).$

   (a) C intercept at (0,0)

   (b) t intercepts at (0,0), (3,0) (1,0)

7. Zeros: $x \approx -1.65, \ x \approx 3.64, \ x \approx 5.$

9. (a) as $t \to \infty, h(t) \to \infty.$

   (b) as $t \to -\infty, h(t) \to -\infty$

For part a of problem 9, we see that as soon as t becomes greater than 5, the function $h(t) =$ $3(t - 5)^3(t - 3)^3(t - 2)$ will increase positively as it approaches infinity, because as soon as t is greater than 5, the numbers within each parentheses will always be positive. In b, notice as t approaches -∞, any negative number cubed will stay negative. If you multiply first three terms: $[3 * (t - 5)^3 * (t - 3)^3]$, as t approaches -∞, it will always create a positive number. When you then multiply that by the final number: $(t - 2)$, you will be multiplying a negative: $(t - 2)$, by a positive: $[3 * (t - 5)^3 * (t - 3)^3]$, which will be a negative number.

11. (a) as $t \to \infty, p(t) \to -\infty$

   (b) as $t \to -\infty, p(t) \to -\infty$

For part a of this problem as t approaches positive infinity, you will always have two parts of the equation $p(t) = -2t(t - 1)(3 - t)^2$, that are positive, once t is greater than 1: $[(t - 1) *$ $(3 - t)^2]$, when multiplied together they stay positive. They are then multiplied by a number that will always be negative: -2t. A negative multiplied by a positive is always negative, so p(t) approaches -∞. For part b of this problem, as t approaches negative infinity, you will always have two parts of the equation that are always positive: $[-2t * (3 - t)^2]$, when multiplied together stay positive. They are then multiplied by a number that will always be negative: (t-1). A negative multiplied by a positive is always negative, so p(t) approaches -∞.

13. $f(x) = (x + 3)^2(x - 2)$ 15. $h(x) = (x - 1)^3(x + 3)^2$
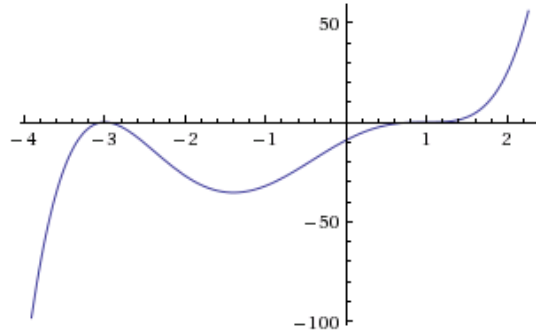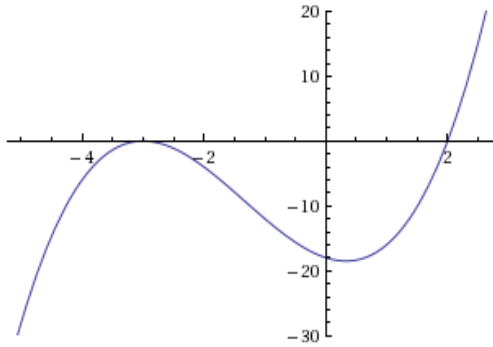
17. $m(x) = -2x(x-1)(x+3)$
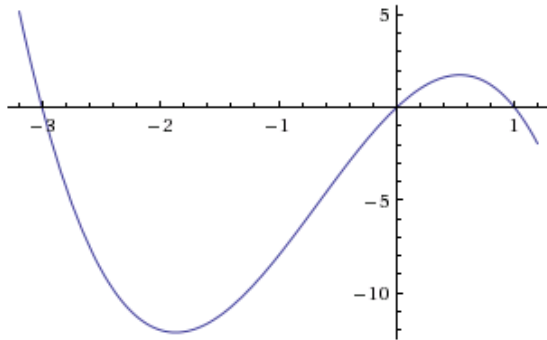
19. $(x-3)(x-2)^2 > 0$ when $x > 3$

To solve the inequality $(x-3)(x-2)^2 > 0$, you first want to solve for x, when the function would be equal to zero. In this case, once you've solved for x, you know that when $f(x) = 0$, $x = 3$, and $x = 2$. You want to test numbers greater than, less than, and in-between these points, to see if these intervals are positive or negative. If an interval is positive it is part of your solution, and if it's negative it's not part of your solution. You test the intervals by plugging any number greater than 3, less than 2, or in between 2 and 3 into your inequality. For this problem, $(x-3)(x-2)^2 > 0$ is only positive when x is greater than 3. So your solution is: $(x - 3)(x-2)^2 > 0$, when $x > 3$.

21. $(x-1)(x+2)(x-3) < 0$ when -2 < x < 1, and when $x > 3$

To solve the inequality $(x-1)(x+2)(x-3) < 0$, you first want to solve for x, when the function would be equal to zero. In this case, once you've solved for x, you know that when $f(x) = 0$, $x = 1$, $x = -2$, and $x = 3$. You want to test numbers greater than, less than, and in-between these points, to see if these intervals are positive or negative. If an interval is positive it is part of your solution, and if it's negative it's not part of your solution. You test the intervals by plugging any number greater than 3, less than -2, or in between -2 and 1, and in between 1 and 3 into your inequality. For this problem, $(x-1)(x+2)(x-3) < 0$ is positive when x is greater than 3, and when it's in between -2 and 1. So your solution is: $(x-1)(x+2)(x-3) < 0$ when $-2 < x < 1$, and when $x > 3$.

23.   The domain is the values of x for which the expression under the radical is nonnegative:

$-42 + 19x - 2x^2 \geq 0$

$-(2x^2 - 19x + 42) \geq 0$

$-(2x - 7)(x - 6) \geq 0$

Recall that this graph is a parabola which opens down, so the nonnegative portion is the interval between (and including) the x-intercepts: $\frac{7}{2} < x < 6$.

25.   The domain is the values of x for which the expression under the radical is nonnegative:

$$4 - 5x - x^2 \geq 0$$
$$(x - 4)(x - 1) \geq 0$$

Recall that this graph is a parabola which opens up, so the nonnegative portions are the intervals outside of (and including) the x-intercepts: $x \leq 1$ and $x \geq 4$.

27.   The domain is the values of x for which the expression under the radical is nonnegative, and since $(x + 2)^2$ is always nonnegative, we need only consider where $x - 3 > 0$, so the domain is $x \geq 3$.

29.   The domain can be any numbers for which the denominator of $p(t)$ is nonzero, because you can't have a zero in the denominator of a fraction. So find what values of t make $t^2 + 2t - 8 = 0$, and those values are not in the domain of $p(t)$. $t^2 + 2t - 8 = (t + 4)(t - 2)$, so the domain is $\mathbb{R}$ where $x \neq -4$ and $x \neq 2$.

31. $f(x) = -\frac{2}{3}(x + 2)(x - 1)(x - 3)$

For problem 31, you can use the x intercepts you're given to get to the point $f(x) = a(x + 2)(x - 1)(x - 3)$, because you know that if you solved for each of the x values you

would end up with the horizontal intercepts given to you in the problem. Since your equation is of degree three, you don't need to raise any of your x values to a power, because if you foiled $(x + 2)(x - 1)(x - 3)$ there will be an x^3, which is degree three. To solve for a, (your stretch factor, in this case $-\frac{2}{3}$), you can plug the point your given, (in this case it's the y intercept $(0, -4)$) into your equation: $-4 = (0 + 2)(0 - 1)(0 - 3)$, to solve for a.

33. $f(x) = \frac{1}{3}(x - 3)^2(x - 1)^2(x + 3)$

For problem 33, you can use the x intercepts you're given to get to the point $f(x) = a(x - 3)^2(x - 1)^2(x + 3)$, because you know that if you solved for each of the x values you would end up with the horizontal intercepts given to you in the problem. The problem tells you at what intercepts has what roots of multiplicity to give a degree of 5, which is why $(x - 2)$ and $(x - 1)$ are squared. To solve for a, (your stretch factor, in this case, $\frac{1}{3}$), you can plug the point your given, (in this case it's the y intercept $(0,9)$) into your equation: $9 = (0 - 3)^2(0 - 1)^2(0 + 3)$, to solve for a.

35. $f(x) = -15(x - 1)^2(x - 3)^3$

For problem 35, you can use the x intercepts you're given to get to the point $f(x) = a(x - 1)^2(x - 3)^3$, because you know that if you solved for each of the x values you would end up with the horizontal intercepts given to you in the problem. The problem tells you at what intercepts has what roots of multiplicity to give a degree of 5, which is why $(x - 1)$ is squared, and $(x - 3)$ is cubed. To solve for a, (your stretch factor, in this case, -15), you can plug the point your given, (in this case it's $(2,15)$) into your equation: $15 = (2 - 1)^2(2 - 3)^3$, to solve for a.

37. The x-intercepts of the graph are (-2, 0), (1, 0), and (3, 0). Then $f(x)$ must include the factors $(x + 2)$, $(x - 1)$, and $(x - 3)$ to ensure that these points are on the graph of $f(x)$, and there cannot be any other factors since the graph has no other x-intercepts. The graph passes through these three x-intercepts without any flattening behavior, so they are single zeros. Filling in what we know so far about the function: $f(x) = a(x + 2)(x - 1)(x - 3)$. To find the value of a, we can use the y-intercept, (0, 3):

$$3 = a(0 + 2)(0 - 1)(0 - 3)$$
$$3 = 6a$$

$$a = \frac{1}{2}$$

Then we conclude that $f(x) = \frac{1}{2}(x + 2)(x - 1)(x - 3)$.

39. $f(x) = -(x + 1)^2(x - 2)$

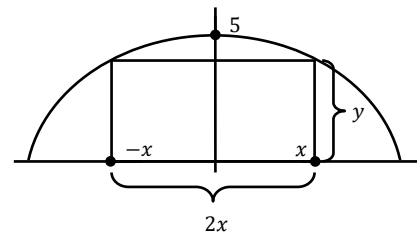41. $f(x) = -\frac{1}{24}(x + 3)(x + 2)(x - 2)(x - 4)$

43. $f(x) = \frac{1}{24}(x + 4)(x + 2)(x - 3)^2$

45. $f(x) = \frac{3}{32}(x + 2)^2(x - 3)^2$

47. $f(x) = \frac{1}{6}(x + 3)(x + 2)(x - 1)^3$

49. $f(x) = -\frac{1}{16}(x + 3)(x + 1)(x - 2)^2(x - 4)$

51. See the diagram below. The area of the rectangle is $A = 2xy$, and $y = 5 - x^2$, so $A = 2x(5 - x^2) = 10x - 2x^3$. Using technology, evaluate the maximum of $10x - 2x^3$. The output value will be maximum area, and the x-value will be half of base width. Plugging the x back into $y = 5 - x^2$ will give the height of the rectangle. The maximum is at $x = 1.29, A = 8.61$. So, Base $= 2.58$, Height $= 3.34$.

problem 51

Section 3.4 Solutions

1. $4x^2 + 3x - 1 = (x - 3)(4x + 15) + 44$

3. $5x^4 - 3x^3 + 2x^2 - 1 = (x^2 + 4)(5x^2 - 3x - 18) + (12x + 71)$

5. $9x^3 + 5 = (2x - 3)\left(\frac{9}{2}x^2 + \frac{27}{4}x + \frac{81}{8}\right) + \frac{283}{8}$

7.

$$
\begin{array}{c|ccc}
1 & 3 & -2 & 1 \\
  & \downarrow & 3 & 1 \\
\hline
  & 3 & 1 & \boxed{2}
\end{array}
$$

$(3x^2 - 2x + 1) = (x - 1)(3x + 1) + 2$

```
-1 |  -2    -4     3
          ↓   2     2
       -2    -2  |  5  |
```
9.
$$(3 - 4x - 2x^2) = (x + 1)(-2x - 2) + 5$$

```
-2 |  1     0     0     8
        ↓   -2     4    -8
     1    -2     4  |  0  |
```
11.
$$(x^3 + 8) = (x + 2)(x^2 - 2x + 4) + 0$$

```
5/3 |  18   -15   -25
          ↓   30    25
       18    15  |  0  |
```
13.
$$(18x^2 - 15x - 25) = \left(x - \tfrac{5}{3}\right)(18x + 15) + 0$$

```
-1/2 |  2     1     2     1
          ↓   -1     0    -1
       2     0     2  |  0  |
```
15.
$$(2x^3 + x^2 + 2x + 1) = \left(x + \tfrac{1}{2}\right)(2x^2 + 2) + 0$$

```
1/2 |  2     0    -3     1
         ↓    1    1/2  -5/4
      2     1   -5/2  | -1/4 |
```
17.
$$(2x^3 - 3x + 1) = \left(x - \tfrac{1}{2}\right)\left(2x^2 + x - \tfrac{5}{2}\right) - \tfrac{1}{4}$$

```
√3 |  1     0    -6     0     9
        ↓   √3     3   -3√3   -9
     1    √3    -3   -3√3 |  0  |
```
19.
$$(x^4 - 6x^2 + 9) = (x - \sqrt{3})(x^3 + \sqrt{3}\,x^2 - 3x - 3\sqrt{3}) + 0$$

21. Dividing by $x - 1$ leaves $x^2 - 5x - 6 = (x - 2)(x - 3)$.
$x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$

23. Dividing by $x - \tfrac{2}{3}$ leaves $3x^2 + 6x + 3 = 3(x^2 + 2x + 1) = 3(x + 1)^2$
$3x^3 + 4x^2 - x - 2 = 3\left(x - \tfrac{2}{3}\right)(x + 1)^2$

25. Dividing by $x + 2$ leaves $x^2 - 3 = (x + \sqrt{3})(x - \sqrt{3})$

$x^3 + 2x^2 - 3x - 6 = (x + 2)(x + \sqrt{3})(x - \sqrt{3})$

27. Dividing by $x - \frac{1}{2}$ twice leaves $4x^2 - 24x + 36 = 4(x^2 - 6x + 9) = 4(x - 3)^2$.

$4x^4 - 28x^3 + 61x^2 - 42x + 9 = 4\left(x - \frac{1}{2}\right)^2 (x - 3)^2$

Section 3.5 Solutions

1. All of the real zeros lie in the interval $[-7,7]$. Possible rational zeros are $\pm 1, \pm 2, \pm 3$

3. All of the real zeros lie in the interval $[-13,13]$

Possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

5. All of the real zeros lie in the interval $[-8,8]$

Possible rational zeros are $\pm 1, \pm 7$

7. All of the real zeros lie in the interval $[-3,3]$

Possible rational zeros are $\pm \frac{1}{17}, \pm \frac{2}{17}, \pm \frac{5}{17}, \pm \frac{10}{17}, \pm 1, \pm 2, \pm 5, \pm 10$

9. All of the real zeros lie in the interval $\left[-\frac{14}{3}, \frac{14}{3}\right]$

Possible rational zeros are $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm 1, \pm 2, \pm 5, \pm 10$

11. Dividing by $x - 1$ leaves $x^2 - x - 6 = (x - 3)(x + 2)$, so
$x^3 - 2x^2 - 5x + 6 = (x - 1)(x - 3)(x + 2)$. Real zeros are

$x = -2, x = 1, x = 3$ (each has mult. 1)

13. Dividing by $x - 1$ leaves $x^3 + x^2 - 8x - 12$.
Dividing again by $x + 2$ leaves $x^2 - x - 6 = (x - 3)(x + 2)$, so
$x^4 - 9x^2 - 4x + 12 = (x - 1)(x + 2)(x - 3)(x + 2)$. Real zeros are
$x = -2$ (mult. 2), $x = 1$ (mult. 1), $x = 3$ (mult. 1)

15. Dividing by $x - 7$ leaves $x^2 + 1$, which is not further reducible.
Real zero is $x = 7$ (mult. 1)

17. Looking at a graph it's clear there are no integer roots. The first positive root looks near $\frac{1}{4}$ or $\frac{1}{3}$. From our list of possible rational zeros (see #7 above), the value near that is $\frac{5}{17}$. Dividing by $x - \frac{5}{17}$ leaves

$-17x^2 + 34 = -17(x^2 - 2) = -17(x + \sqrt{2})(x - \sqrt{2})$. So

$-17x^3 + 5x^2 + 34x - 10 = -17\left(x - \frac{5}{17}\right)(x + \sqrt{2})(x - \sqrt{2})$.

The real zeros are $x = \frac{5}{17}, x = \pm\sqrt{2}$ (each has mult. 1)
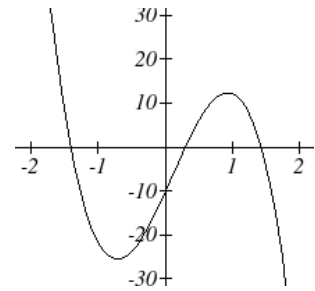
19. Dividing by $x + 2$ leaves $3x^2 - 3x - 5$. This does not factor nicely, but we can use quadratic equation to solve for the roots. The real zeros are

$x = -2, x = \frac{3\pm\sqrt{69}}{6}$ (each has mult. 1)

21. We can factor out the common factor x, giving $x(9x^2 - 5x - 1)$. The quadratic does not factor nicely, but we can use quadratic equation. The real zeros are

$x = 0, x = \frac{5\pm\sqrt{61}}{18}$ (each has mult. 1)

23. We can factor this as $(x^2 + 5)(x^2 - 3)$. $x^2 + 5 = 0$ has no real solutions, but $x^2 - 3 = 0$ gives the real zeros: $x = \pm\sqrt{3}$ (each has mult. 1)

25. This can factor as $(3x^2 + 1)(x^2 - 5)$. $3x^2 + 1$ has no real solutions, but $x^2 - 5 = 0$ gives the real zeros: $x = \pm\sqrt{5}$ (each has mult. 1)

27. This can factor as $(x^3 - 5)(x^3 + 2)$. $x = \sqrt[3]{-2} = -\sqrt[3]{2}, x = \sqrt[3]{5}$ (each has mult. 1)

29. Dividing by $x - 2$ leaves $x^4 - 4 = (x^2 + 2)(x^2 - 2)$. $x^2 + 2$ gives no real solutions, but $x^2 - 2 = 0$ gives $x = \pm\sqrt{2}$. The real zeros are: $x = 2, x = \pm\sqrt{2}$ (each has mult. 1)

31. $x = -4$ (mult. 3), $x = 6$ (mult. 2)

Section 3.6 Solutions

1. $3i$

3. -12

5. $1 + \sqrt{3}i$

7. $8 - i$

9. $-11 + 4i$

11. $8x + 12i^2 = -12 + 8i$

13. $30 - 10i$

15. $8 - 2i + 12i - 3i^2 = 11 + 10i$

17. $16 + 8i - 8i - 4i^2 = 20$

19. $\frac{3}{2} + 2i$

21. $\frac{(-5+3i)}{2i} \cdot \frac{i}{i} = \frac{-5i-3}{-2} = \frac{3}{2} + \frac{5}{2}i$

23. $\dfrac{2-3i}{4+3i} \cdot \dfrac{4-3i}{4-3i} = \dfrac{8-6i-12i+9i^2}{16-12i+12i-9i^2} = \dfrac{-1-18i}{25} = -\dfrac{1}{25} - \dfrac{18}{25}i$

25. Using quadratic formula, $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} = \dfrac{4 \pm \sqrt{-36}}{2} = \dfrac{4 \pm 6i}{2} = 2 \pm 3i$.

$f(x) = x^2 - 4x + 13 = (x - (2 + 3i))(x - (2 - 3i))$.

27. Using quadratic formula, $x = \dfrac{-2 \pm \sqrt{2^2 - 4(3)(10)}}{2(3)} = \dfrac{-2 \pm \sqrt{-116}}{6} = \dfrac{-2 \pm 2\sqrt{29}i}{6} = -\dfrac{1}{3} \pm \dfrac{\sqrt{29}}{3}i$.

$f(x) = 3x^2 + 2x + 10 = 3\left(x - \left(-\dfrac{1}{3} + \dfrac{\sqrt{29}}{3}i\right)\right)\left(x - \left(-\dfrac{1}{3} - \dfrac{\sqrt{29}}{3}i\right)\right)$.

29. Dividing by $x + 5$ leaves $x^2 + x + 1$. $x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1 \pm \sqrt{3}i}{2} = -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i$

$f(x) = x^3 + 6x^2 + 6x + 5 = (x + 5)\left(x - \left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)\right)\left(x - \left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right)\right)$

Zeros: $x = -5, \ x = -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i$

31. Dividing by $x + 3$ leaves $x^2 + 4$. $x^2 + 4 = 0$ gives $x^2 = -4$, so $x = \pm\sqrt{-4} = \pm 2i$.

$f(x) = x^3 + 3x^2 + 4x + 12 = (x + 3)(x^2 + 4) = (x + 3)(x + 2i)(x - 2i)$.

Zeros: $x = -3, \ \pm 2i$

33. $f(x) = x^3 + 7x^2 + 9x - 2 = (x + 2)\left(x - \left(-\dfrac{5}{2} + \dfrac{\sqrt{29}}{2}\right)\right)\left(x - \left(-\dfrac{5}{2} - \dfrac{\sqrt{29}}{2}\right)\right)$

Zeros: $x = -2, \ x = -\dfrac{5}{2} \pm \dfrac{\sqrt{29}}{2}$

35. $f(x) = 4x^4 - 4x^3 + 13x^2 - 12x + 3 = \left(x - \dfrac{1}{2}\right)^2 (4x^2 + 12)$

$= 4\left(x - \dfrac{1}{2}\right)^2 (x + i\sqrt{3})(x - i\sqrt{3})$

Zeros: $x = \dfrac{1}{2}, \ x = \pm\sqrt{3}i$

37. $f(x) = x^4 + x^3 + 7x^2 + 9x - 18 = (x + 2)(x - 1)(x^2 + 9)$

$= (x + 2)(x - 1)(x + 3i)(x - 3i)$

Zeros: $x = -2, \ 1, \ \pm 3i$

39. $f(x) = -3x^4 - 8x^3 - 12x^2 - 12x - 5 = (x + 1)^2(-3x^2 - 2x - 5)$

$= -3(x + 1)^2\left(x - \left(-\dfrac{1}{3} + \dfrac{\sqrt{14}}{3}i\right)\right)\left(x - \left(-\dfrac{1}{3} - \dfrac{\sqrt{14}}{3}i\right)\right)$

Zeros: $x = -1, \ x = -\dfrac{1}{3} \pm \dfrac{\sqrt{14}}{3}i$

41. $f(x) = x^4 + 9x^2 + 20 = (x^2 + 4)(x^2 + 5) = (x - 2i)(x + 2i)(x - i\sqrt{5})(x + i\sqrt{5})$

Zeros: $x = \pm 2i, \pm i\sqrt{5}$

Section 3.7 Solutions

1. b   3. a

5. Vertical asymptote: $x = -4$, Horizontal asymptote: $y = 2$, Vertical intercept: $(0, -\frac{3}{4})$,

Horizontal intercept: $(\frac{3}{2}, 0)$. For problem 5, the vertical asymptote is $x = -4$ because that gives

a 0 in the denominator, which makes the function undefined. The horizontal asymptote is $y = 2$,

because since the degrees are equal in the numerator, and denominator, the ratio of the leading

coefficients the answer. The vertical intercept is $(0, -\frac{3}{4})$ because $f(0) = -\frac{3}{4}$, and the

horizontal intercept is $(\frac{3}{2}, 0)$, because when the function $p(x) = 0$, $x = \frac{3}{2}$.

7. Vertical asymptote: $x = 2$, Horizontal asymptote: $y = 0$, Vertical intercept: $(0, 1)$,

Horizontal intercept: none. For this problem, the vertical asymptote is $x = 2$ because $x = 2$,

gives a 0 in the denominator, which is undefined. The horizontal asymptote is $y = 0$, because

there is only an x in the denominator, so as $\rightarrow \infty$, $f(x) = 0$. The vertical intercept is $(0, 1)$

because $f(0) = 1$, and there is no horizontal intercept because when $f(x) = 0$, there is no

solution for x.

9. Vertical asymptotes: $= -4, \frac{4}{3}$, Horizontal asymptote: $y = 1$, Vertical intercept:

$(0, \frac{5}{16})$, Horizontal intercepts: $\left(-\frac{1}{3}, 0\right), (5, 0)$. There are two vertical asymptotes because the

denominator is equal to 0 for two different values of x. The horizontal asymptote is $y = 1$,

because when the degrees in the numerator and denominator are equal, the ratio of their

coefficients is 1. The vertical intercept is $\frac{5}{16}$, because $f(0) = \frac{5}{16}$, and the horizontal intercepts

are $-\frac{1}{3}$, 5 because when $f(x) = 0$, x can equal both $-\frac{1}{3}$, and 5.

11. Vertical asymptote: there is no vertical asymptote, Horizontal asymptote: $y = 1$, Vertical intercept: $(0, 3)$, Horizontal intercept: $(-3, 0)$. Since $x = 1$ gives a 0 in both the numerator and the denominator, it does not give a vertical asymptote for problem 11. The horizontal asymptote is $y = 1$, because since the degrees are equal in the numerator, and denominator, the leading coefficient is the answer. The vertical intercept is (0, 3) because $f(0) = 3$, and the horizontal intercept is (-3, 0), because when the function $f(x) = 0, x = -3$.

13. Vertical asymptote: $x = 3$, Horizontal asymptote: There is no horizontal asymptote, Vertical intercept: $(0, \frac{1}{4})$, Horizontal intercepts: $(-1, 0), (\frac{1}{2}, 0)$. For problem 13, the vertical asymptote is $x = 3$ because that gives a 0 in the denominator, which makes the function undefined. Since the degree in the numerator than the degree in the denominator there is no horizontal asymptote, because as $x \to \infty, f(x) \to \infty$. The vertical intercept is $(0, \frac{1}{4})$, because $f(0) = \frac{1}{4}$, and the horizontal intercepts are $(-1, 0), (\frac{1}{2}, 0)$, because when the function f=0, x=-1 or $\frac{1}{2}$.

15. Vertical asymptotes: x = 0, and x = 4, Horizontal asymptote: y = 0, Vertical intercept: none, Horizontal intercepts: $(-2, 0), (\frac{2}{3}, 0)$. For problem 15, the vertical asymptotes are at $x =$ 0 and $x = 4$, because that is where the denominator becomes zero and the function becomes undefined. The horizontal asymptote is 0 because the highest degree in the denominator is bigger than the highest degree in the numerator. The vertical intercept is undefined because $f(0) = -4/0$, which is undefined, and the horizontal intercepts are $(-2, 0), (\frac{2}{3}, 0)$, because when $f(x) = 0, x = -2, 2/3$.

17. Vertical asymptotes: $x = -2, 4$, Horizontal asymptote: $y = 1$, Vertical intercept: $(0, -\frac{15}{16})$, Horizontal intercepts, (1, 0), (-3, 0), and (5, 0). The numerator and denominator are already given in factored form, making it easier to find the vertical asymptotes and horizontal intercepts. Computing $w(0)$ gives the vertical intercept. For the horizontal asymptote, observe that if the numerator and denominator were each multiplied out, they'd both have a leading term of x^3, so they each have the same degree and the leading coefficient 1, giving the horizontal asymptote $y = 1$.

19. $(x + 1)$ gives us a zero at $x = -1$, $(x - 2)$ gives us a zero at $x = 2$. $(x - 5)$ and $(x + 5)$ give us vertical asymptotes at 5 and -5 respectively, because these values give us undefined terms. Using this we then plug $x = 0$ into our equation which gives us $-\frac{2}{25}$ however, we need this to equal 4, so multiplying by 50 does the trick. $y = \frac{50(x-2)(x+1)}{(x-5)(x+5)}$

21. Refer to problem 19. To get 7, we look at the long run behavior of y as $x \to \infty$. If we expand the numerator and denominator of our function we get that they both have degree 2. Then we get a horizontal asymptote at 1, so multiplying our function by 7 gives us a horizontal asymptote at 7. $y = \frac{7(x-4)(x+6)}{(x+4)(x+5)}$

23. See problem 21 and/or problem 19. To get a double zero at $x = 2$, we need the numerator to be able to be broken down into two factors both of which are zero at $x = 2$. So, $(x - 2)^2$ appears in the numerator.

25. . This graph has vertical asymptotes at $x = -3$ and $x = 4$, which gives us our denominator. The function's only zero is at $x = 3$, so we get an $(x - 3)$ term in our numerator. Evaluating our function so far, $y = \frac{(x-3)}{(x-4)(x+3)}$, at zero we get $\frac{1}{4}$, but the graph has $y = 1$ at $x = 0$, so multiplying the function by 4 gives us our desired result, $y = \frac{4(x-3)}{(x-4)(x+3)}$

27. $y = -\frac{9(x-2)}{(x-3)(x+3)}$ see problem 25.

29. $y = \frac{(x-2)(x+3)}{3(x-1)}$ This function has zeros at -3 and 2, so this gives us a numerator of $(x + 3)(x - 2)$. We have a vertical asymptote at $x = 1$ so we get a $(x - 1)$ term in the denominator. Then evaluating our function so far at zero, we get 6, so including a 3 in our denominator gives us our desired result of having $y(0) = 2$.

31. $y = \frac{3(x-1)^2}{(x-2)(x+3)}$ This function has a zero at $x = 1$ and vertical asymptotes at $x = 2$ and $x = -3$. However, we need to have an increasing function in the region $-3 < x < 1$ and $\frac{(x-1)}{(x-2)(x+3)}$ is decreasing in this region. If we square our numerator we get this outcome without changing any

of our zeros or vertical asymptotes. Then we multiply our new function by 3 to get our correct y-intercept.

33. $y = -\dfrac{2x(x-3)}{(x-4)(x+3)}$ see problem 31

35. $y = \dfrac{(x-1)^3}{(x+1)(x-2)^2}$ Knowing that we have vertical asymptotes at $x = -1$ and $x = 2$, we get that we need an $(x + 1)$ factor and a $(x - 2)$ factor in our denominator. Knowing that we have a zero at $x = 1$, we know our denominator is made up of $(x - 1)$ terms. To get the function that we want, we know we need a horizontal asymptote at 2 so our degree of our numerator and denominator must match. Experimenting with different powers we get $(x - 1)^3$ and $(x + 1)(x - 2)^2$.

37. $y = \dfrac{-(x-4)3}{(x-4)(x+1)} + 1$. To get a "hole" or a non-value at $x = 4$, having a $\dfrac{(x-4)}{(x-4)}$ term will cancel everywhere with the exception of $x = 4$ where the function will be undefined. Then, knowing that there is a vertical asymptote at $x = -1$ we have a $(x + 1)$ term in the denominator. Multiplying by (-3) and then adding 1 gives our desired shifts.

39. Long division gives $f(x) = 3x - 2 + \dfrac{4}{x+2}$. The oblique asymptote is $y = 3x - 2$.

41. Long division gives $f(x) = \dfrac{1}{2}x + 1 + \dfrac{3}{2x-6}$. The oblique asymptote is $y = \dfrac{1}{2}x + 1$.

43. Long division gives $f(x) = -2x + 1 + \dfrac{4}{x^2+3}$. The oblique asymptote is $y = -2x + 1$.

45. (a) To get the percentage of water (non-acid) in the beaker we take $(n + 16)$ our total amount of water and divide by our total amount of solution $(n + 20)$. Then to get the percent of acid, we subtract the percent of water from 1, or $1 - \dfrac{(n+16)}{(n+20)}$.

(b) Using our equation from part (a), we get $1 - \dfrac{(10+16)}{(10+20)} = 13.33\%$

(c) To get 4%, we use our equation from part (a) to solve: $4 = 1 - \dfrac{n+16}{n+20} \rightarrow -3 = \dfrac{(n+16)}{(n+20)} \rightarrow$

$-3n - 60 = n + 16 \rightarrow n = 19mL$.

(d)   As $n \to \infty$, $\frac{n+16}{n+20} \to 1$, because the denominator and numerator have the same degree. So,

as $\to \infty$, $1 - \frac{n+16}{n+20} \to 0$. This means that our acid becomes insignificant compared to the water.

47. (a) We are given the form $m(x) = \frac{ax+b}{cx+d}$ for the equation, so we need to find values for

$a, b, c,$ and d. Notice that there is more than one correct solution, since multiplying the

numerator and denominator by the same value gives an equivalent function (for example,

multiplying by, say, $\frac{3}{3}$, does not change the value of an expression). This means we are free to

choose a value for one of the unknown numbers, but once that's chosen, the values of the other

three must follow from the given information. For this solution, let's start by choosing $c = 1$.

We are told that when Oscar is "far down the hallway", the meter reads 0.2. The word "far"

implies that this is the long-run behavior, so the horizontal asymptote is $y = 0.2$. For the given

form of $m(x)$, the horizontal asymptote has the form $y = \frac{a}{c}$, so $\frac{a}{c} = 0.2$. Since we previously

decided to let $c = 1$, we now know $a = 0.2$. Other language in the problem implies that $m(6) =$

2.3 and $m(8) = 4.4$ (assuming $x = 0$ at the entrance to the room). We can plug in these values

for x and $m(x)$ to get the equations: $2.3 = \frac{0.2*6+b}{6+d}$ and $4.4 = \frac{0.2*8+b}{8+d}$. Simplify these equations

by multiplying both sides of each by the respective denominators. Solve the resulting system of

equations (perhaps using substitution), which gives $b = -10.4$ and $d = -10$, so the equation is

$m(x) = \frac{0.2x-10.4}{x-10}$. (As previously stated, answers may vary, so long as the equations are

equivalent. For example, a second possible solution is $m(x) = \frac{x-52}{5x-50}$. To show this is

equivalent, multiply the previous answer by $\frac{5}{5}$.)

(b) The given values 10 and 100 are meter readings, so substitute them for $m(x)$ and solve for x.
When $m(x) = 10$:

$$10 = \frac{0.2x - 10.4}{x - 10}$$

$$10x - 100 = 0.2x - 10.4$$

$$9.8x = 89.6$$

$$x \approx 9.143$$

The algebra is similar when $m(x) = 100$, yielding $x \approx 9.916$.

(c) The meter reading increases as Oscar gets closer to the magnet. The graph of the function for (a) has a vertical asymptote at $x = 10$, for which the graph shoots up when approaching from the left, so the magnet is 10 feet into the room. This is consistent with our answers for (b), in which x gets closer to 10 as the meter reading increases.

49. (a)   To solve for k, we plug $c = 1$ and $d = 20$ into our equation $c = \dfrac{k}{d^2}$ and we get $k = 400$.

(b)   To get from 15 miles/hour to feet/sec we use the following calculation:

$(15)\left(\dfrac{1\ mi}{1}\right)\left(\dfrac{5280ft}{1mi}\right)\left(\dfrac{1}{hour}\right)\left(\dfrac{1hour}{60min}\right)\left(\dfrac{1min}{60sec}\right) = 22\dfrac{ft}{sec}$. The distance of Olav from point a is $(33 - 22t)$. Then, our total distance to the stoplight is $d = \sqrt{10^2 + (33 - 22t)^2}$ which simplifies to $d = \sqrt{1189 - 1452t + 484t^2}$. So, our function is $C(t) = \dfrac{400}{1189 - 1452t + 484t^2}$.

(c)   The light will shine on Olav the brightest when he has travelled 33 feet. To solve for what time this will happen, we solve the equation $22t = 33$ for t. So, the light will be the brightest at 1.5 seconds.

(d) $2 = \dfrac{400}{1189 - 1452t + 484t^2}$ to solve for t we get $200 = 1189 - 1452t + 484t^2$ or $0 = 989 - 1452t + 484t^2$. Using the quadratic formula, we get $t = \dfrac{23}{22}$ and $\dfrac{43}{22}$.

3.8 Solutions to Exercises

1. Domain: $x \geq 4$. By determining the vertex of this transformed function as (4,0), we know x has to be $x \geq 4$ for f(x) to be a one-to-one non-decreasing function. To find the inverse:

$$y = (x - 4)^2$$
$$\pm\sqrt{y} = x - 4$$
$$-4 \pm \sqrt{y} = x$$

Since we restricted the domain of the function to $x \geq 4$, the range of the inverse function should be the same, telling us to use the positive case. So $f^{-1}(y) = -4 + \sqrt{y}$.

3. Domain: $x \leq 0$, because this parabola opens down with a vertex of (0, 12). To find the inverse:

$$y = 12 - x^2$$
$$x^2 = 12 - y$$
$$x = \pm\sqrt{12 - y}$$

Since we restricted the domain of the function to $x \leq 0$, the range of the inverse function should be the same, telling us to use the negative case. So $f^{-1}(y) = -\sqrt{12 - y}$.

5. Domain: all real numbers, because this function is always one-to-one and increasing. To find the inverse:

$$y = 3x^3 + 1$$
$$y - 1 = 3x^3$$
$$\frac{y-1}{3} = x^3$$
$$\cdot \quad x = f^{-1}(y) = \sqrt[3]{\frac{y-1}{3}}$$

7. $y = 9 + \sqrt{4x - 4}$

$y - 9 = \sqrt{4x - 4}$

$(y - 9)^2 = 4x - 4$

$(y - 9)^2 = 4x - 4$

$(y - 9)^2 + 4 = 4x$

$x = f^{-1}(y) = \frac{(y-9)^2}{4} + 1$

9. $y = 9 + 2\sqrt[3]{x}$

$y - 9 = 2\sqrt[3]{x}$

$\frac{y-9}{2} = \sqrt[3]{x}$

$x = f^{-1}(y) = \left(\frac{y-9}{2}\right)^3$

11. $y = \frac{2}{x+8}$

$y(x + 8) = 2$

$xy + 8y = 2$

$x = f^{-1}(x) = \frac{2-8y}{y}$

13. $y = \frac{x+3}{x+7}$

$y(x + 7) = x + 3$

$xy + 7y = x + 3$

$xy - x = 3 - 7y$

$x(y - 1) = 3 - 7y$

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$$x = f^{-1}(x) = \frac{3-7y}{y-1}$$

15. Using the same algebraic methods as Problem 13, we get $f^{-1}(y) = \frac{5y-4}{4y+3}$.

17. $v \approx 65.57 \ mph$. This problem is asking for the speed (v) given the length (l) so you can plug in 215 for L and solve.

19. $v \approx 34.07 \ mph$. Refer to problem 17 (with r for radius instead of l for length)

21. Impose a coordinate system with the origin at the bottom of the ditch. Then the parabola will be in the form $y = ax^2$, and the points on either side at the top of the ditch are (-10, 10) and (10, 10). Plugging either into the general form and solving for a gives $a = 0.1$. To find the x-coordinates where the water meets the edges of the ditch, plug 5 into $y = 0.1x^2$ for y. Solving for x gives approximately 7.07. Note that this is just half of the width of the surface of the water, so the entire width is about 14.14 feet.

23. (a) $h = -2x^2 + 124x$ Since the slope of the cliff is -4, the equation of the cliff is $y = -4x$. Then the equations relating the height h of the rocket above the sloping ground is the height of the ground subtracted from the height of the rocket.

(b) The maximum height of the rocket over the ground is at the vertex of the parabola. To find the vertex you first want to solve for the x coordinate of the vertex: $\left(-\frac{b}{2a}\right)$. So $h = -2x^2 + 124x$, then $a = -2$, $b = 124$, so $\left(-\frac{124}{2(-2)}\right) = x = 31$. To find the vertex you can then plug in your x coordinate of the vertex into your function: $h = -2(31)^2 + 124(31) =$ 1922 feet. So the rocket's maximum height above the ground is 1922 feet.

(c) To find a function for $x = g(h)$, solve $h = -2x^2 + 124x$ for x:

$h = -2x^2 + 124x$

$2x^2 - 124x + h = 0$

$x = \frac{124 \pm \sqrt{15{,}376 - 8h}}{4}$ by the quadratic formula

$x = g(h) = \frac{62 - \sqrt{3{,}844 - 2h}}{2}$ by reducing and taking the negative root in the numerator to give the lower of the two possible x-coordinates, since the rocket is going up on the left half of the parabola.

    (d) The function given in (c) does not work when the function is going down. We would have had to choose the positive root in the numerator to give the right half of the parabola.