Chapter 3 Exercises

1. Let A(x) represent the area bounded by the graph and the horizontal axis and vertical lines at t=0 and t=x for the graph in Fig. 33. Evaluate A(x) for x = 1, 2, 3, 4, and 5.

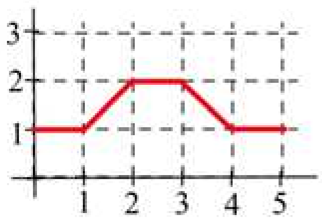

Figure 1

2. Let B(x) represent the area bounded by the graph and the horizontal axis and vertical lines at t=0 and t=x for the graph in Fig. 34. Evaluate B(x) for x = 1, 2, 3, 4, and 5.

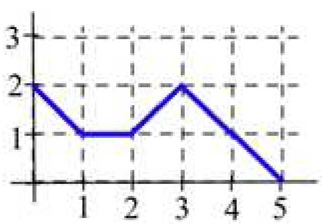

Figure 2

3. Let C(x) represent the area bounded by the graph and the horizontal axis and vertical lines at t=0 and t=x for the graph in Fig. 35. Evaluate C(x) for x = 1, 2, and 3 and find a formula for C(x).

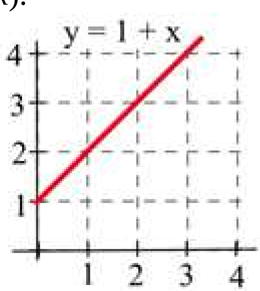

Figure 3

4. Let A(x) represent the area bounded by the graph and the horizontal axis and vertical lines at t=0 and t=x for the graph in Fig. 36. Evaluate A(x) for x = 1, 2, and 3 and find a formula for A(x).

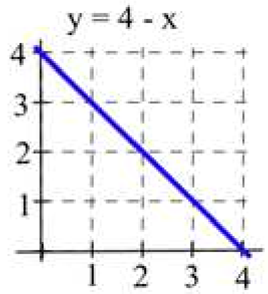

Figure 4

5. A car had the velocity shown in Fig. 37. How far did the car travel from t= 0 to t = 30 seconds?

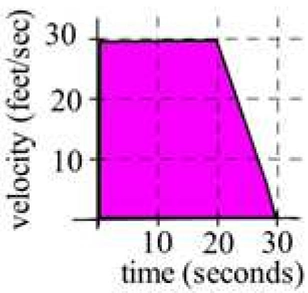

Figure 5

6. A car had the velocity shown in Fig. 38. How afar did the car travel from t = 0 to t = 30 seconds?

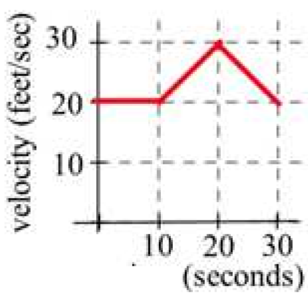

Figure 6

7. The velocities of two cars are shown in Fig. 39.
(a) From the time the brakes were applied, how many seconds did it take each car to stop?
(b) From the time the brakes were applied, which car traveled farther until it came to a complete stop?

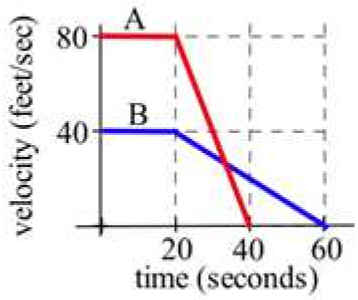

Figure 7

8. You and a friend start off at noon and walk in the same direction along the same path at the rates shown in Fig. 40.
(a) Who is walking faster at 2 pm? Who is ahead at 2 pm?
(b) Who is walking faster at 3 pm? Who is ahead at 3 pm?
(c) When will you and your friend be together? (Answer in words.)

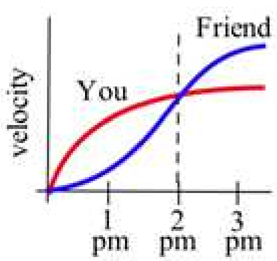

Figure 8

9. Police chase: A speeder traveling 45 miles per hour (in a 25 mph zone) passes a stopped police car which immediately takes off after the speeder. If the police car speeds up steadily to 60 miles/hour in 20 seconds and then travels at a steady 60 miles/hour, **how long** and **how far** before the police car catches the speeder who continued traveling at 45 miles/hour? (Fig. 41)

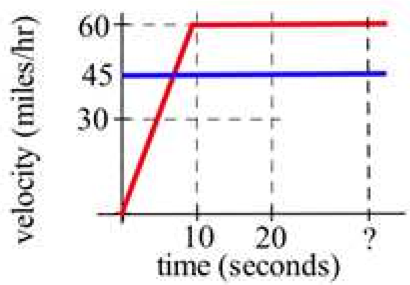

Figure 9

10. Water is flowing into a tub. The table shows the rate at which the water flows, in gallons per minute. The tub is initially empty.

t, in minutes	0	1	2	3	4	5	6	7	8	9	10
Flow rate, in gal/min	0.5	1.0	1.2	1.4	1.7	2.0	2.3	1.8	0.7	0.5	0.2

Use the table to estimate how much water is in the tub after

a. five minutes

b. ten minutes

11. The table shows the speedometer readings for a short car trip.

t , in minutes	0	5	10	15	20
Speed, in mph	0	30	40	65	40

a. Use the table to estimate how far the car traveled over the twenty minutes shown.

b. How accurate would you expect your estimate to be?

12. The table shows values of $f(t)$. Use the table to estimate $\int_0^{40} f(t)\,dt$.

t	0	10	20	30	40
$f(t)$	17	22	18	11	35

13. The table shows values of $g(x)$.

x	0	1	2	3	4	5	6
$g(x)$	140	142	144	152	154	165	200

Use the table to estimate

a. $\int_0^3 g(x)\,dx$

b. $\int_3^6 g(x)\,dx$

c. $\int_0^6 g(x)\,dx$

14. What are the units for the "area" of a rectangle with the given base and height units?

Base units	Height units	"Area" units
miles per second	seconds	
hours	dollars per hour	
square feet	feet	
kilowatts	hours	
houses	people per house	
meals	meals	

In problems 15 – 17 , represent the area of each bounded region as a definite integral, and use geometry to
determine the value of the definite integral.

15. The region bounded by $y = 2x$, the x–axis, the line $x = 1$, and $x = 3$.

16. The region bounded by $y = 4 - 2x$, the x–axis, and the y–axis.

17. The shaded region in Fig. 42.

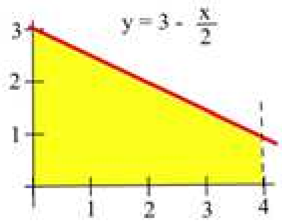

Figure 10

18. Fig. 43 shows the graph of f and the areas of several regions. Evaluate:

(a) $\int_0^3 f(x)\,dx$ (b) $\int_3^5 f(x)\,dx$ (c) $\int_3^7 f(x)\,dx$

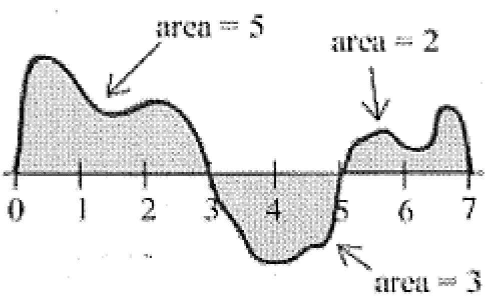

Figure 11

19. Fig. 44 shows the graph of g and the areas of several regions.

Evaluate : (a) $\int_{1}^{3} g(x)\,dx$ (b) $\int_{3}^{4} g(x)\,dx$

(c) $\int_{4}^{8} g(x)\,dx$ (d) $\int_{1}^{8} g(x)\,dx$ (e) $\int_{3}^{8} |\,g(x)\,|\,dx$

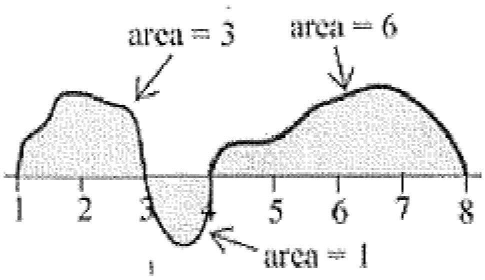

Figure 12

20. Fig. 45 shows the graph of h. Use the graph to evaluate:

(a) $\int_{-2}^{1} h(x)\,dx$ (b) $\int_{4}^{6} h(x)\,dx$ (c) $\int_{-2}^{6} h(x)\,dx$ (d) $\int_{-2}^{4} h(x)\,dx$

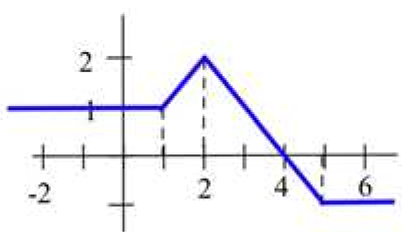

Figure 13

21. Your velocity along a straight road is shown in Fig. 46. How far did you travel in 8 minutes?

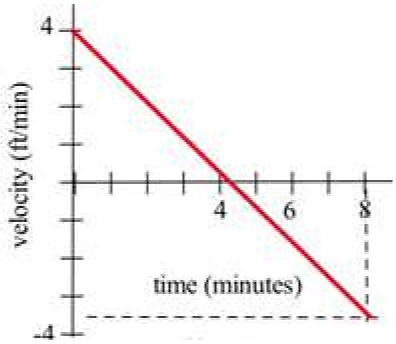

Figure 14

22. Your velocity along a straight road is shown in Fig. 47. How many feet did you walk in 8 minutes?

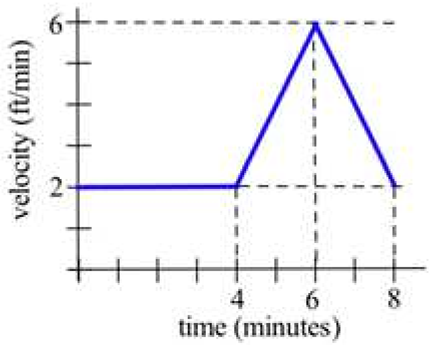

Figure 15

In problems 23 - 26, the units are given for x and for f(x) . Give the units of $\int_a^b f(x)\,dx$.

23. x is time in "seconds", and f(x) is velocity in "meters per second."

24. x is time in "hours", and f(x) is a flow rate in "gallons per hour."

25. x is a position in "feet", and f(x) is an area in "square feet."

26. x is a position in "inches", and f(x) is a density in "pounds per inch."

In problems 27 – 31, represent the area with a definite integral and use technology to find the approximate answer.

27. The region bounded by $y = x^3$, the x–axis, the line $x = 1$, and $x = 5$.

28. The region bounded by $y = \sqrt{x}$, the x–axis, and the line $x = 9$.

29. The shaded region in Fig. 48.

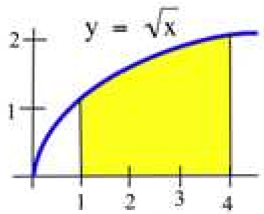

Figure 16

30. The shaded region in Fig. 49.

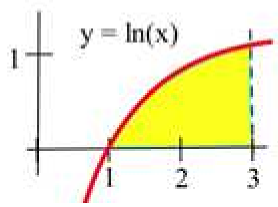

Figure 17

31. The shaded region in Fig. 49 for $2 \le x \le 3$.

32. Consider the definite integral $\int_0^3 (3 + x)\,dx$.

(a) Using six rectangles, find the left-hand Riemann sum for this definite integral.
(b) Using six rectangles, find the right-hand Riemann sum for this definite integral.
(c) Using geometry, find the exact value of this definite integral.

33. Consider the definite integral $\int_0^2 x^3\, dx$.

(a) Using four rectangles, find the left-hand Riemann sum for this definite integral.
(b) Using four rectangles, find the right-hand Riemann sum for this definite integral.

Problems 34 – 41 refer to the graph of f in Fig. 50. Use the graph to determine the values of the definite integrals. (The bold numbers represent the **area** of each region.)

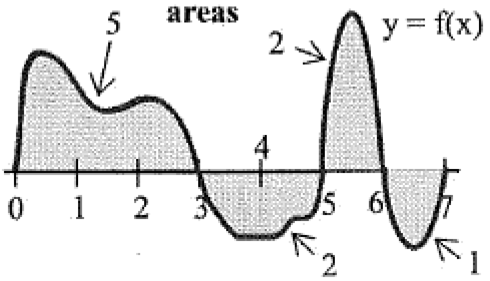

Figure 18

34. $\int_0^3 f(x)\, dx$ **35.** $\int_3^5 f(x)\, dx$ **36.** $\int_2^2 f(x)\, dx$ **37.** $\int_6^7 f(w)\, dw$ **38.** $\int_0^5 f(x)\, dx$

39. $\int_0^7 f(x)\, dx$ **40.** $\int_3^6 f(t)\, dt$ **41.** $\int_5^7 f(x)\, dx$

Problems 42 – 47 refer to the graph of g in Fig. 51. Use the graph to evaluate the integrals.

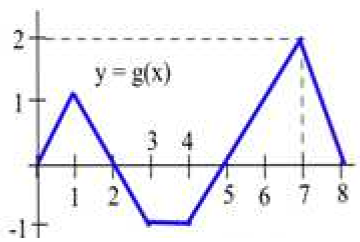

Figure 19

42. $\int_0^2 g(x)\, dx$ **43.** $\int_1^3 g(t)\, dt$ **44.** $\int_0^5 g(x)\, dx$

45. $\int_0^8 g(s)\,ds$ **46.** $\int_0^3 2g(t)\,dt$ **47.** $\int_5^8 1+g(x)\,dx$

48. Write the total distance traveled by the car in Fig. 52 between 1 pm and 4 pm as a definite integral and estimate the value of the integral.

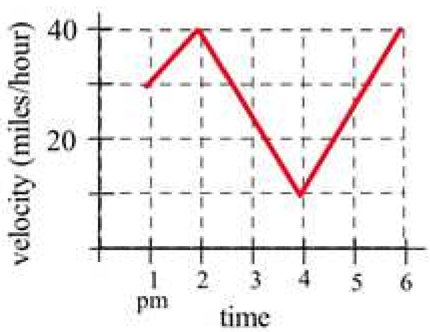

Figure 20

49. Write the total distance traveled by the car in Fig. 52 between 3 pm and 6 pm as a definite integral and estimate the value of the integral.

For problems 50 - 67, find the indicated antiderivative.

50. $\int \left(x^3 - 14x + 5\right)dx$

51. $\int \left(2.5x^5 - x - 1.25\right)dx$

52. $\int 12.3dy$

53. $\int \pi^2 dw$

54. $\int e^P dP$

55. $\int \left(\sqrt{x} + e^x - \frac{1}{4x^3}\right)dx$

56. $\int \frac{1}{x} dx$

57. $\int \frac{1}{x^2} dx$

58. $\int (x-2)(x+2)dx$

59. $\int \frac{t^5 - t^2}{t} dt$

60. $\int \frac{1}{(4x+1)^3} dx$

61. $\int e^{100x} dx$

62. $\int (1.0003)^{12t} dt$

63. $\int \frac{e^{10/x}}{x^2} dx$

64. $\int \sqrt{w+5}\, dw$

65. $\int 6x^2 \sqrt{3x^3 - 1}\, dx$

66. $\int \frac{dx}{x \ln x}$

67. $\int \frac{x-3}{x^2 - 6x + 5} dx$

For problems 68 - 79, find an antiderivative of the integrand and use the Fundamental Theorem to evaluate the definite integral.

68. $\int_{2}^{5} 3x^2 \, dx$

69. $\int_{-1}^{2} x^2 \, dx$

70. $\int_{1}^{3} (x^2 + 4x - 3) \, dx$

71. $\int_{1}^{e} \frac{1}{x} \, dx$

72. $\int_{25}^{100} \sqrt{x}\, dx$

73. $\int_{3}^{5} \sqrt{x}\, dx$

74. $\int_{1}^{10} \frac{1}{x^2} dx$

75. $\int_{1}^{1000} \frac{1}{x^2} dx$

76. $\int_{0}^{1} e^x dx$

77. $\int_{-2}^{2} \frac{2x}{1+x^2} dx$

78. $\int_{0}^{1} e^{2x} \, dx$

79. $\int_{2}^{4} (x-2)^3 \, dx$

80. Find the area shown in Fig. 53

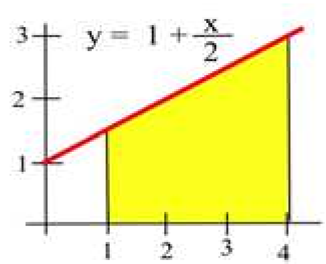

Figure 21

81. Find the area shown in Fig. 54

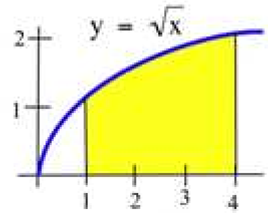

Figure 22

82. Find the area shown in Fig. 55

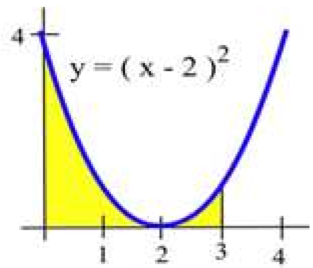

Figure 23

83. Find the area shown in Fig. 56

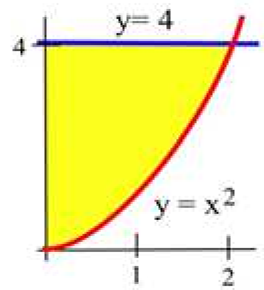

Figure 24

In problems 84 – 87, use the values in the table to estimate the areas.

x	$f(x)$	$g(x)$	$h(x)$
0	5	2	5
1	6	1	6
2	6	2	8
3	4	2	6
4	3	3	5
5	2	4	4
6	2	0	2

84. Estimate the area between *f* and *g*, between $x = 0$ and $x = 4$.
85. Estimate the area between *g* and *h*, between $x = 0$ and $x = 6$.
86. Estimate the area between *f* and *h*, between $x = 0$ and $x = 4$.
87. Estimate the area between *f* and *g*, between $x = 0$ and $x = 6$.

88. Estimate the area of the island in Fig. 57.

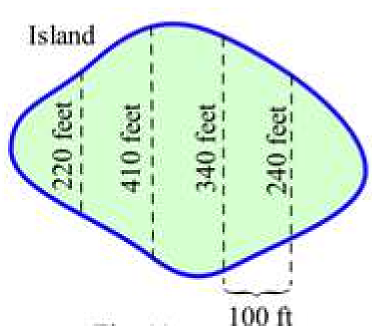

Figure 25

In problems 89 – 98, find the **area** between the graphs of f and g for x in the given interval. Remember to draw the graph!

89. $f(x) = x^2 + 3$, $g(x) = 1$ and $-1 \le x \le 2$.

90. $f(x) = x^2 + 3$, $g(x) = 1 + x$ and $0 \le x \le 3$.

91. $f(x) = x^2$, $g(x) = x$ and $0 \le x \le 2$.

92. $f(x) = (x-1)^2$, $g(x) = x + 1$ and $0 \le x \le 3$.

93. $f(x) = \frac{1}{x}$, $g(x) = x$ and $1 \le x \le e$.

94. $f(x) = \sqrt{x}$, $g(x) = x$ and $0 \le x \le 4$.

95. $f(x) = 4 - x^2$, $g(x) = x + 2$ and $0 \le x \le 2$.

96. $f(x) = e^x$, $g(x) = x$ and $0 \le x \le 2$.

97. $f(x) = 3$, $g(x) = \sqrt{1 - x^2}$ and $0 \le x \le 1$.

98. $f(x) = 2$, $g(x) = \sqrt{4 - x^2}$ and $-2 \le x \le 2$.

In problems 99 and 100, use the values in the table to estimate the average values.

x	$f(x)$	$g(x)$
0	5	2
1	6	1
2	6	2
3	4	2
4	3	3
5	2	4
6	2	0

99. Estimate the average value of f on the interval [0, 6].

100. Estimate the average value of g on the interval [0, 6].

In problems 101 – 106, find the **average value** of f on the given interval.

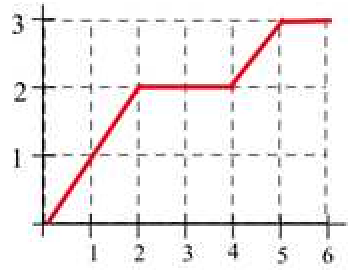

Figure 26

101. f(x) in Fig. 58 for $0 \le x \le 2$.

102. f(x) in Fig. 58 for $0 \le x \le 4$.

103. f(x) in Fig. 58 for $1 \le x \le 6$.

104. f(x) in Fig. 58 for $4 \le x \le 6$.

105. $f(x) = 2x + 1$ for $0 \le x \le 4$.

106. $f(x) = x^2$ for $0 \le x \le 2$.

107. Fig. 59 shows the velocity of a car during a 5 hour trip.
(a) Estimate how far the car traveled during the 5 hours.
(b) At what **constant** velocity should you drive in order to travel the same distance in 5 hours?

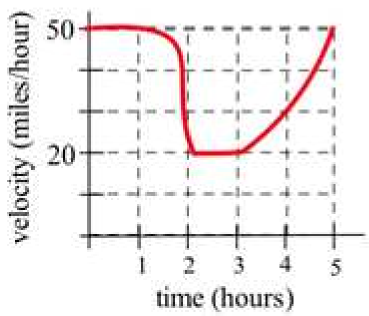

Figure 27

108. Fig. 60 shows the number of telephone calls per minute at a large company.
(a) Estimate the average number of calls per minute from 8 am to 5 pm.
(b) From 9 am to 1 pm.

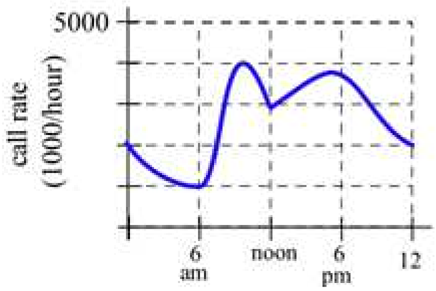

Figure 28

109. The demand and supply functions for a certain product are given by $p = 150 - .5q$ and $p = .002q^2 + 1.5$, where p is in dollars and q is the number of items.
(a) Which is the demand function?
(b) Find the equilibrium price and quantity
(c) Find the total gains from trade at the equilibrium price.

110. Still thinking about the product from Exercise 109, with its demand and supply functions, suppose the price is set artificially at $70 (which is above the equilibrium price).
(a) Find the quantity supplied and the quantity demanded at this price.
(b) Compute the consumer surplus at this price, using the quantity demanded.
(c) Compute the producer surplus at this price, using the quantity demanded (why?).
(d) Find the total gains from trade at this price.
(e) What do you observe?

111. When the price of a certain product is $40, 25 items can be sold. When the price of the same product costs $20, 185 items can be sold. On the other hand, when the price of this product is $40, 200 items will be produced. But when the price of this product is $20, only 100 items will be produced. Use this information to find supply and demand functions (assume for simplicity that the functions are linear), and compute the consumer and producer surplus at the equilibrium price.

112. Find the present and future values of a continuous income stream of $5000 per year for 12 years if money can earn 1.3% annual interest compounded continuously.

113. Find the present value of a continuous income stream of \$40,000 per year for 35 years if money can earn
(a) 0.8% annual interest, compounded continuously,
(b) 2.5% annual interest, compounded continuously,
(c) 4.5% annual interest, compounded continuously.

114. Find the present value of a continuous income stream $F(t) = 20 + t$, where *t* is in years and *F* is in tens of thousands of dollars per year, for 10 years, if money can earn 2% annual interest, compounded continuously.

115. Find the present value of a continuous income stream $F(t) = 12 + 0.3t^2$, where *t* is in years and *F* is in thousands of dollars per year, for 8 years, if money can earn 3.7% annual interest, compounded continuously.

116. Find the future value of a continuous income stream $F(t) = 8500 + \sqrt{640t + 100}$, where *t* is in years and *F* is in dollars per year, for 15 years, if money can earn 6% annual interest, compounded continuously.

117. A business is expected to generate income at a continuous rate of \$25,000 per year for the next eight years. Money can earn 3.4% annual interest, compounded continuously. The business is for sale for \$153,000. Is this a good deal?