5.1.1 Integers

Learning Objective(s)

1 Locate integers on a number line.
2 Find the absolute value of a given number.
3 Find the opposite of a given number.

Introduction

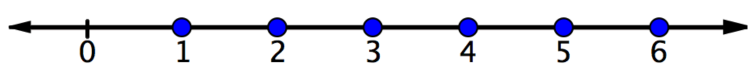
You've worked with numbers on a number line. You know how to graph numbers like 0, 1, 2, 3, etc. on the number line. There are other kinds of numbers that can be graphed on the number line, too**.** Let's see what they look like and where they are located on the number line.

Natural Numbers and Whole Numbers

In mathematics, it's sometimes helpful to talk about groups of things, which are called **sets**. Numbers can be grouped into sets, and a particular number can belong to more than one set.

You probably are familiar with the set of **natural numbers**, which are also called the **counting numbers**. These are the numbers 1, 2, 3, and so on—the numbers we use when *counting.*

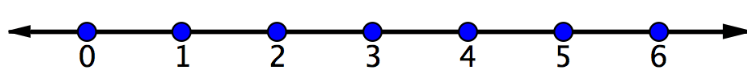
The following illustration shows the natural numbers graphed on a number line.



The number line continues in both directions. The set of natural numbers only continues to the right, so you can include 6, 7, and so on, all the way up into the hundreds, thousands, and beyond. You can only show so much on one picture!

When 0 is added to the set of 1, 2, 3, and so on, it forms the set of **whole numbers**. These are called "whole" because they have no fractional parts. (A trick to help you remember which are *natural* numbers and which are *whole* numbers is to think of a "hole," which can be represented by 0. The whole ("hole") numbers include 0, the natural numbers do not.)

The following illustration shows the whole numbers graphed on the number line.



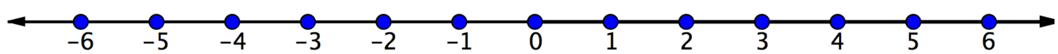
Integers

Objective **1**

When you work with something like temperature, you sometimes want to use numbers that are less than zero, which are called **negative numbers**. Negative numbers are written using a negative sign in front, such as −1, −5, and −30. These are read "negative one," "negative five," and "negative thirty." (The negative sign should not be read as "minus"; *minus* means subtraction.)

The numbers greater than 0 are called **positive numbers** and can be written with or without the "+" sign. Notice that 0 is neither positive nor negative!

Integers are the numbers: ..., −3, −2, −1, 0, 1, 2, 3, ... Notice that all of the whole numbers are also integers. The following illustration shows the integers graphed on the number line. The integers include zero and continue to the right and to the left.



Self Check A

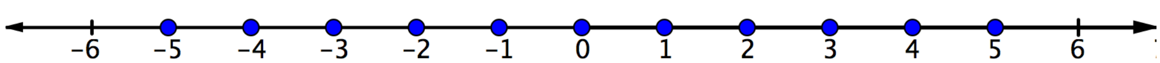
The number 0 belongs to which of the following sets of numbers?

natural numbers
whole numbers
integers

Absolute Value and the Number Line

Objective **2**

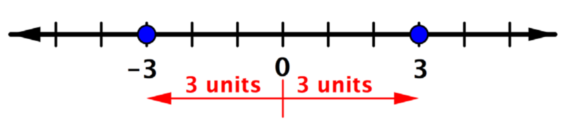
The number line below shows all the integers between and including −5 and 5. Notice that the positive integers go to the right: 1, 2, 3, and so on. The negative integers go to the left: −1, −2, −3, and so on.



The distance between a number's place on the number line and 0 is called the number's **absolute value**. To write the absolute value of a number, use short vertical lines (|) on either side of the number. For example, the absolute value of −3 is written |−3|.

Notice that distance is always positive or 0.

|−3| = 3, as −3 is 3 units away from 0 and |3| = 3, as 3 is 3 units away from 0.



Here are some other examples.

$|0| = 0$

$|-23| = 23$

$|6| = 6$

$|817| = 817$

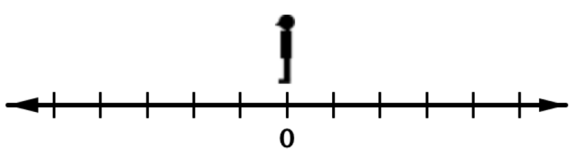
$|-3{,}000| = 3{,}000$

Example		
Problem	**Find |-7|**	
Answer	$\|-7\| = 7$	Since −7 is 7 units from 0, the absolute value is 7.

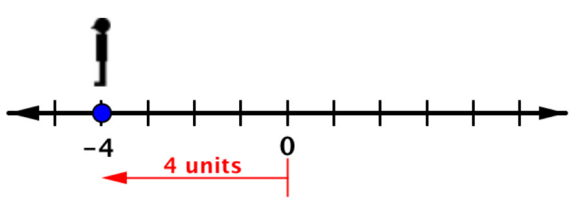
To locate an integer on the number line, imagine standing on the number line at 0. If the number is 0, you're there. If the number is positive, face to the right—numbers greater than 0. If the number is negative, face to the left—numbers less than 0. Then, move forward the number of units equal to the absolute value of the number.

Example

Problem **Find −4 on the number line. Then determine |−4|.**



Imagine standing at 0. Since −4 is negative, face to the left.

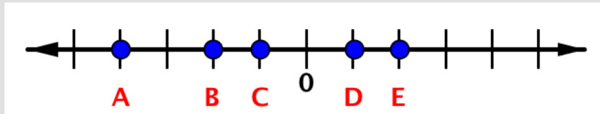


Move 4 units from 0 in the negative direction.

Draw a dot on the number line at that location, which is −4.

Answer	$\|-4\| = 4$	Direction moved does not affect absolute value, only the distance moved.

Self Check B
Which point represents −2 on this number line?



Opposites

Objective 3

You may have noticed that, except for 0, the integers come in pairs of positive and negative numbers: 1 and −1, 3 and −3, 72 and −72, and so on. Each number is the **opposite** of the other number in the pair: 72 is the opposite of −72, and −72 is the opposite of 72.

A number and its opposite are the same distance from 0, so they have the same absolute value.

$$|72| = 72, \text{ and } |-72| = 72$$

The set of **integers** are all the whole numbers and their opposites.

Self Check C
What is the opposite of −29?

Summary

Some numbers are natural numbers (1, 2, 3, ...) or whole numbers (0, 1, 2, 3, ...). Whole numbers are also integers. There are other integers that are the opposites of the whole numbers (−1, −2, −3, ...). These negative numbers lie to the left of 0 on the number line. Integers are the whole numbers and their opposites. The absolute value of a number is its distance to 0 on the number line. Absolute values are always positive or 0.

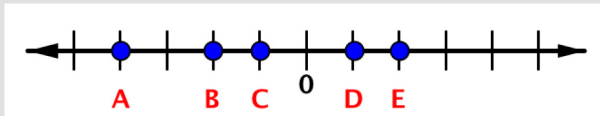
5.1.1 Self Check Solutions

Self Check A
The number 0 belongs to which of the following sets of numbers?

Both whole numbers and integers include 0, but the natural numbers do not.

Self Check B

Which point represents −2 on this number line?



Point B is 2 units to the left of 0, so it represents −2.

Self Check C

What is the opposite of −29?

Answer: The opposite of −29 is 29.

5.1.2 Rational and Real Numbers

Learning Objective(s)

1 Identify the subset(s) of the real numbers that a given number belongs to.
2 Locate points on a number line.
3 Compare rational numbers.
4 Identify rational and irrational numbers.

Introduction

You've worked with fractions and decimals, like 3.8 and $21\frac{2}{3}$. These numbers can be found between the integer numbers on a number line. There are other numbers that can be found on a number line, too. When you include all the numbers that can be put on a number line, you have the real number line. Let's dig deeper into the number line and see what those numbers look like. Let's take a closer look to see where these numbers fall on the number line.

Rational Numbers

Objective **1, 2**

The fraction $\frac{16}{3}$, mixed number $5\frac{1}{3}$, and decimal 5.33… (or $5.\overline{3}$) all represent the same number. This number belongs to a set of numbers that mathematicians call **rational numbers**. Rational numbers are numbers that can be written as a ratio of two integers. Regardless of the form used, $5.\overline{3}$ is rational because this number *can* be written as the ratio of 16 over 3, or $\frac{16}{3}$.

Examples of rational numbers include the following.

0.5, as it can be written as $\frac{1}{2}$

$2\frac{3}{4}$, as it can be written as $\frac{11}{4}$

−1.6, as it can be written as $-1\frac{6}{10} = \frac{-16}{10}$

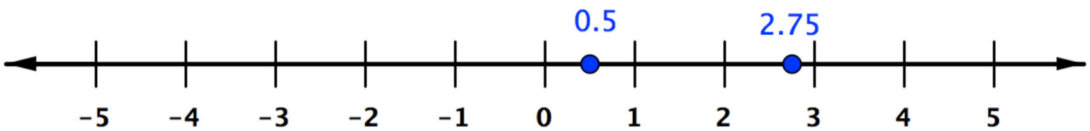
4, as it can be written as $\frac{4}{1}$

-10, as it can be written as $\frac{-10}{1}$

All of these numbers can be written as the ratio of two integers.

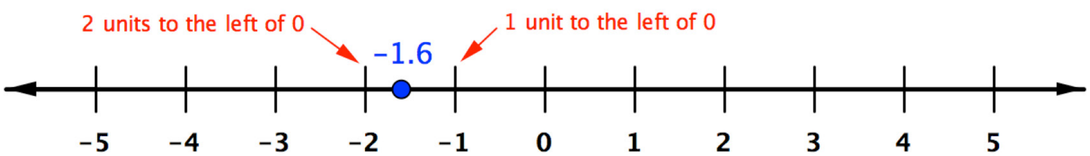
You can locate these points on the number line.

In the following illustration, points are shown for 0.5 or $\frac{1}{2}$, and for 2.75 or $2\frac{3}{4} = \frac{11}{4}$.



As you have seen, rational numbers can be negative. Each positive rational number has an opposite. The opposite of $5.\overline{3}$ is $-5.\overline{3}$, for example.

Be careful when placing **negative numbers** on a number line. The negative sign means the number is to the left of 0, and the absolute value of the number is the distance from 0. So to place −1.6 on a number line, you would find a point that is |−1.6| or 1.6 units to the left of 0. This is more than 1 unit away, but less than 2.



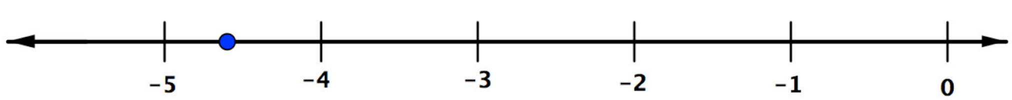
Example

Problem **Place $-\frac{23}{5}$ on a number line.**

It's helpful to first write this improper fraction as a mixed number: 23 divided by 5 is 4 with a remainder of 3, so $-\frac{23}{5}$ is $-4\frac{3}{5}$.

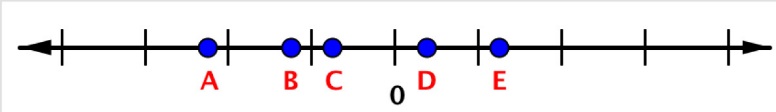
Since the number is negative, you can think of it as moving $4\frac{3}{5}$ units to the *left* of 0. $-4\frac{3}{5}$ will be between −4 and −5.

Answer



Self Check A

Which of the following points represents $-1\frac{1}{4}$?



Comparing Rational Numbers

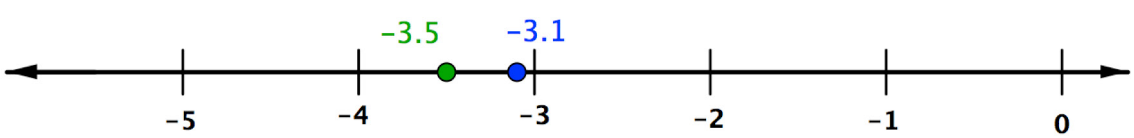
Objective 3

When two **whole numbers** are graphed on a number line, the number to the right on the number line is always greater than the number on the left.

The same is true when comparing two **integers** or rational numbers. The number to the right on the number line is always greater than the one on the left.

Here are some examples.

Numbers to Compare	Comparison	Symbolic Expression
−2 and −3	−2 is greater than −3 because −2 is to the right of −3	$-2 > -3$ or $-3 < -2$
2 and 3	3 is greater than 2 because 3 is to the right of 2	$3 > 2$ or $2 < 3$
−3.5 and −3.1	−3.1 is greater than −3.5 because −3.1 is to the right of −3.5 (see below)	$-3.1 > -3.5$ or $-3.5 < -3.1$



Self Check B

Which of the following are true?

i. $-4.1 > 3.2$
ii. $-3.2 > -4.1$
iii. $3.2 > 4.1$
iv. $-4.6 < -4.1$

Irrational and Real Numbers

Objective **4**

There are also numbers that are not rational. **Irrational numbers** cannot be written as the ratio of two integers.

Any square root of a number that is not a perfect square, for example $\sqrt{2}$, is irrational. Irrational numbers are most commonly written in one of three ways: as a root (such as a square root), using a special symbol (such as π), or as a nonrepeating, nonterminating decimal.

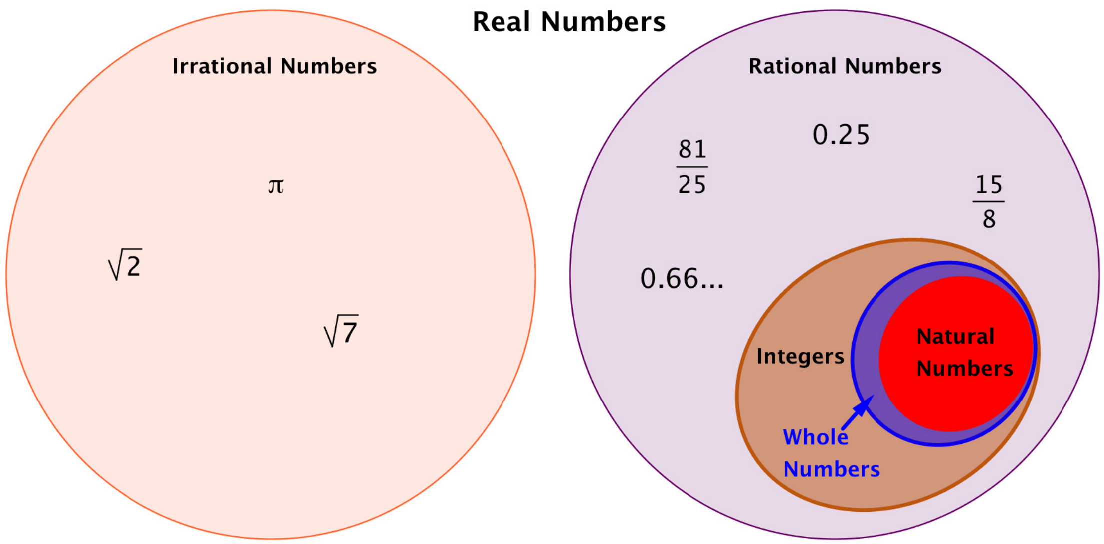
Numbers with a decimal part can either be **terminating decimals** or **nonterminating decimals**. Terminating means the digits stop eventually (although you can always write 0s at the end). For example, 1.3 is terminating, because there's a last digit. The decimal form of $\frac{1}{4}$ is 0.25. Terminating decimals are always rational.

Nonterminating decimals have digits (other than 0) that continue forever. For example, consider the decimal form of $\frac{1}{3}$, which is 0.3333…. The 3s continue indefinitely. Or the decimal form of $\frac{1}{11}$, which is 0.090909…: the sequence "09" continues forever.

In addition to being nonterminating, these two numbers are also **repeating decimals**. Their decimal parts are made of a number or sequence of numbers that repeats again and again. A **nonrepeating decimal** has digits that never form a repeating pattern. The value of $\sqrt{2}$, for example, is 1.414213562…. No matter how far you carry out the numbers, the digits will never repeat a previous sequence.

If a number is terminating *or* repeating, it must be rational; if it is both nonterminating *and* nonrepeating, the number is irrational.

Type of Decimal	Rational or Irrational	Examples
Terminating	Rational	0.25 (or $\frac{1}{4}$) 1.3 (or $\frac{13}{10}$)
Nonterminating and Repeating	Rational	0.66… (or $\frac{2}{3}$) 3.242424… (or) $\frac{321}{99}=\frac{107}{33}$
Nonterminating and Nonrepeating	Irrational	π (or 3.14159…) $\sqrt{7}$ (or 2.6457…)



Example	
Problem	**Is −82.91 rational or irrational?**
Answer	−82.91 is rational, because it is a *terminating* decimal.

The **set** of **real numbers** is made by combining the set of rational numbers and the set of irrational numbers. The real numbers include **natural numbers** or **counting numbers**, whole numbers, integers, rational numbers (fractions and repeating or terminating decimals), and irrational numbers. The set of real numbers is all the numbers that have a location on the number line.

Sets of Numbers	
Natural numbers	1, 2, 3, …
Whole numbers	0, 1, 2, 3, …
Integers	…, −3, −2, −1, 0, 1, 2, 3, …
Rational numbers	numbers that can be written as a ratio of two integers—rational numbers are terminating or repeating when written in decimal form
Irrational numbers	numbers than cannot be written as a ratio of two integers—irrational numbers are nonterminating and nonrepeating when written in decimal form
Real numbers	any number that is rational or irrational

Example		
Problem	**What sets of numbers does 32 belong to?**	
Answer	The number 32 belongs to all these sets of numbers: Natural numbers Whole numbers Integers Rational numbers Real numbers	Every natural or counting number belongs to all of these sets!

Example		
Problem	**What sets of numbers does $382.\overline{3}$ belong to?**	
Answer	$382.\overline{3}$ belongs to these sets of numbers: Rational numbers Real numbers	The number is rational because it's a repeating decimal. It's equal to $382\frac{1}{3}$ or $\frac{1,147}{3}$, or $382.\overline{3}$.

Example		
Problem	**What sets of numbers does $-\sqrt{5}$ belong to?**	
Answer	$-\sqrt{5}$ belongs to these sets of numbers: Irrational numbers Real numbers	The number is irrational because it can't be written as a ratio of two integers. Square roots that aren't perfect squares are always irrational.

Self Check C

Which of the following sets does $\frac{-33}{5}$ belong to?

whole numbers
integers
rational numbers
irrational numbers
real numbers

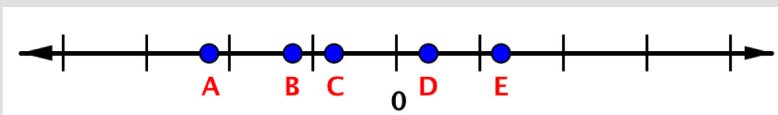
Summary

The set of real numbers is all numbers that can be shown on a number line. This includes natural or counting numbers, whole numbers, and integers. It also includes rational numbers, which are numbers that can be written as a ratio of two integers, and irrational numbers, which cannot be written as a the ratio of two integers. When comparing two numbers, the one with the greater value would appear on the number line to the right of the other one.

5.1.2 Self Check Solutions

Self Check A

Which of the following points represents $-1\frac{1}{4}$?



B.

Negative numbers are to the left of 0, and $-1\frac{1}{4}$ should be 1.25 units to the left. Point B is the only point that's more than 1 unit and less than 2 units to the left of 0.

Self Check B

Which of the following are true?

i. $-4.1 > 3.2$
ii. $-3.2 > -4.1$
iii. $3.2 > 4.1$
iv. $-4.6 < -4.1$

ii and iv
-3.2 is to the right of -4.1, so $-3.2 > -4.1$. Also, -4.6 is to the left of -4.1, so $-4.6 < -4.1$.

Self Check C

Which of the following sets does $\frac{-33}{5}$ belong to?

rational and real numbers
The number is between integers, so it can't be an integer or a whole number. It's written as a ratio of two integers, so it's a rational number and not irrational. All rational numbers are real numbers, so this number is rational and real.