Chapter 4: Exponential and Logarithmic Functions

Section 4.1 Exponential Functions

India is the second most populous country in the world, with a population in 2008 of about 1.14 billion people. The population is growing by about 1.34% each year[1]. We might ask if we can find a formula to model the population, P, as a function of time, t, in years after 2008, if the population continues to grow at this rate.

In linear growth, we had a constant rate of change – a constant *number* that the output increased for each increase in input. For example, in the equation $f(x) = 3x + 4$, the slope tells us the output increases by three each time the input increases by one. This population scenario is different – we have a *percent* rate of change rather than a constant number of people as our rate of change.

To see the significance of this difference consider these two companies:

Company *A* has 100 stores, and expands by opening 50 new stores a year

Company *B* has 100 stores, and expands by increasing the number of stores by 50% of their total each year.

Looking at a few years of growth for these companies:

---

[1] World Bank, World Development Indicators, as reported on http://www.google.com/publicdata, retrieved August 20, 2010

Year	Stores, company A		Stores, company B
0	100	Starting with 100 each	100
1	$100 + 50 = 150$	They both grow by 50 stores in the first year.	$100 + 50\%$ of 100 $100 + 0.50(100) = 150$
2	$150 + 50 = 200$	Store A grows by 50, Store B grows by 75	$150 + 50\%$ of 150 $150 + 0.50(150) = 225$
3	$200 + 50 = 250$	Store A grows by 50, Store B grows by 112.5	$225 + 50\%$ of 225 $225 + 0.50(225) = 337.5$

Notice that with the percent growth, each year the company grows by 50% of the current year's total, so as the company grows larger, the number of stores added in a year grows as well.

To try to simplify the calculations, notice that after 1 year the number of stores for company B was:
$100 + 0.50(100)$ or equivalently by factoring
$100(1+0.50) = 150$

We can think of this as "the new number of stores is the original 100% plus another 50%".

After 2 years, the number of stores was:
$150 + 0.50(150)$ or equivalently by factoring
$150(1+0.50)$ now recall the 150 came from 100(1+0.50). Substituting that,
$100(1+0.50)(1+0.50) = 100(1+0.50)^2 = 225$

After 3 years, the number of stores was:
$225 + 0.50(225)$ or equivalently by factoring
$225(1+0.50)$ now recall the 225 came from $100(1+0.50)^2$. Substituting that,
$100(1+0.50)^2(1+0.50) = 100(1+0.50)^3 = 337.5$

From this, we can generalize, noticing that to show a 50% increase, each year we multiply by a factor of (1+0.50), so after n years, our equation would be
$B(n) = 100(1+0.50)^n$

In this equation, the 100 represented the initial quantity, and the 0.50 was the percent growth rate. Generalizing further, we arrive at the general form of exponential functions.

> **Exponential Function**
>
> An **exponential growth or decay function** is a function that grows or shrinks at a constant percent growth rate. The equation can be written in the form
>
> $$f(x) = a(1+r)^x \quad \text{or} \quad f(x) = ab^x \quad \text{where } b = 1+r$$
>
> Where
>
> a is the initial or starting value of the function
>
> r is the percent growth or decay rate, written as a decimal
>
> b is the growth factor or growth multiplier. Since powers of negative numbers behave strangely, we limit b to positive values.

To see more clearly the difference between exponential and linear growth, compare the two tables and graphs below, which illustrate the growth of company A and B described above over a longer time frame if the growth patterns were to continue.

years	Company A	Company B
2	200	225
4	300	506
6	400	1139
8	500	2563
10	600	5767
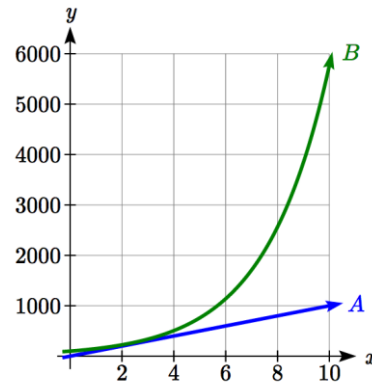
Example 1

Write an exponential function for India's population, and use it to predict the population in 2020.

At the beginning of the chapter we were given India's population of 1.14 billion in the year 2008 and a percent growth rate of 1.34%. Using 2008 as our starting time ($t = 0$), our initial population will be 1.14 billion. Since the percent growth rate was 1.34%, our value for r is 0.0134.

Using the basic formula for exponential growth $f(x) = a(1+r)^x$ we can write the formula, $f(t) = 1.14(1+0.0134)^t$

To estimate the population in 2020, we evaluate the function at $t = 12$, since 2020 is 12 years after 2008.

$f(12) = 1.14(1+0.0134)^{12} \approx 1.337$ billion people in 2020

For comparison, the actual population of India in 2020 was about 1.380 billion.

Try it Now
1. Given the three statements below, identify which represent exponential functions.

A. The cost of living allowance for state employees increases salaries by 3.1% each year.
B. State employees can expect a $300 raise each year they work for the state.
C. Tuition costs have increased by 2.8% each year for the last 3 years.

Example 2
A certificate of deposit (CD) is a type of savings account offered by banks, typically offering a higher interest rate in return for a fixed length of time you will leave your money invested. If a bank offers a 24 month CD with an annual interest rate of 1.2% compounded monthly, how much will a $1000 investment grow to over those 24 months?

First, we must notice that the interest rate is an annual rate, but is compounded monthly, meaning interest is calculated and added to the account monthly. To find the monthly interest rate, we divide the annual rate of 1.2% by 12 since there are 12 months in a year: 1.2%/12 = 0.1%. Each month we will earn 0.1% interest. From this, we can set up an exponential function, with our initial amount of $1000 and a growth rate of $r = 0.001$, and our input m measured in months.

$$f(m) = 1000\left(1 + \frac{.012}{12}\right)^m$$

$$f(m) = 1000(1 + 0.001)^m$$

After 24 months, the account will have grown to $f(24) = 1000(1+0.001)^{24} = \$1024.28$

Try it Now
2. Looking at these two equations that represent the balance in two different savings accounts, which account is growing faster, and which account will have a higher balance after 3 years?

$$A(t) = 1000(1.05)^t \qquad B(t) = 900(1.075)^t$$

In all the preceding examples, we saw exponential growth. Exponential functions can also be used to model quantities that are decreasing at a constant percent rate. An example of this is radioactive decay, a process in which radioactive isotopes of certain atoms transform to an atom of a different type, causing a percentage decrease of the original material over time.

Example 3

Bismuth-210 is an isotope that radioactively decays by about 13% each day, meaning 13% of the remaining Bismuth-210 transforms into another atom (polonium-210 in this case) each day. If you begin with 100 mg of Bismuth-210, how much remains after one week?

With radioactive decay, instead of the quantity increasing at a percent rate, the quantity is decreasing at a percent rate. Our initial quantity is a = 100 mg, and our growth rate will be negative 13%, since we are decreasing: r = -0.13. This gives the equation:
$$Q(d) = 100(1 - 0.13)^d = 100(0.87)^d$$
This can also be explained by recognizing that if 13% decays, then 87 % remains.

After one week, 7 days, the quantity remaining would be
$$Q(7) = 100(0.87)^7 = 37.73 \text{ mg of Bismuth-210 remains.}$$

Try it Now

3. A population of 1000 is decreasing 3% each year. Find the population in 30 years.

Example 4

$T(q)$ represents the total number of Android smart phone contracts, in thousands, held by a certain Verizon store region measured quarterly since January 1, 2016, Interpret all the parts of the equation $T(2) = 86(1.64)^2 = 231.3056$.

Interpreting this from the basic exponential form, we know that 86 is our initial value. This means that on Jan. 1, 2016 this region had 86,000 Android smart phone contracts. Since b = 1 + r = 1.64, we know that every quarter the number of smart phone contracts grows by 64%. $T(2)$ = 231.3056 means that in the 2nd quarter (or at the end of the second quarter) there were approximately 231,306 Android smart phone contracts.

Finding Equations of Exponential Functions

In the previous examples, we were able to write equations for exponential functions since we knew the initial quantity and the growth rate. If we do not know the growth rate, but instead know only some input and output pairs of values, we can still construct an exponential function.

Example 5

In 2009, 80 deer were reintroduced into a wildlife refuge area from which the population had previously been hunted to elimination. By 2015, the population had grown to 180 deer. If this population grows exponentially, find a formula for the function.

By defining our input variable to be t, years after 2009, the information listed can be written as two input-output pairs: $(0,80)$ and $(6,180)$. Notice that by choosing our input variable to be measured as years after the first year value provided, we have effectively "given" ourselves the initial value for the function: $a = 80$. This gives us an equation of the form

$$f(t) = 80b^t.$$

Substituting in our second input-output pair allows us to solve for b:

$180 = 80b^6$ Divide by 80

$b^6 = \dfrac{180}{80} = \dfrac{9}{4}$ Take the 6th root of both sides.

$b = \sqrt[6]{\dfrac{9}{4}} = 1.1447$

This gives us our equation for the population:

$$f(t) = 80(1.1447)^t$$

Recall that since $b = 1+r$, we can interpret this to mean that the population growth rate is $r = 0.1447$, and so the population is growing by about 14.47% each year.

In this example, you could also have used $(9/4)^{\wedge}(1/6)$ to evaluate the 6th root if your calculator doesn't have an nth root button.

In the previous example, we chose to use the $f(x) = ab^x$ form of the exponential function rather than the $f(x) = a(1+r)^x$ form. This choice was entirely arbitrary – either form would be fine to use.

When finding equations, the value for b or r will usually have to be rounded to be written easily. To preserve accuracy, it is important to not over-round these values. Typically, you want to be sure to preserve at least 3 significant digits in the growth rate. For example, if your value for b was 1.00317643, you would want to round this no further than to 1.00318.

In the previous example, we were able to "give" ourselves the initial value by clever definition of our input variable. Next, we consider a situation where we can't do this.

Example 6

Find a formula for an exponential function passing through the points (-2,6) and (2,1).

Since we don't have the initial value, we will take a general approach that will work for any function form with unknown parameters: we will substitute in both given input-output pairs in the function form $f(x) = ab^x$ and solve for the unknown values, a and b.

Substituting in (-2, 6) gives $6 = ab^{-2}$

Substituting in (2, 1) gives $1 = ab^2$

We now solve these as a system of equations. To do so, we could try a substitution approach, solving one equation for a variable, then substituting that expression into the second equation.

Solving $6 = ab^{-2}$ for a:

$$a = \frac{6}{b^{-2}} = 6b^2$$

In the second equation, $1 = ab^2$, we substitute the expression above for a:

$$1 = (6b^2)b^2$$

$$1 = 6b^4$$

$$\frac{1}{6} = b^4$$

$$b = \sqrt[4]{\frac{1}{6}} \approx 0.6389$$

Going back to the equation $a = 6b^2$ lets us find a:

$$a = 6b^2 = 6(0.6389)^2 = 2.4492$$

Putting this together gives the equation $f(x) = 2.4492(0.6389)^x$

Try it Now

4. Given the two points (1, 3) and (2, 4.5) find the equation of an exponential function that passes through these two points.

Example 7

Find an equation for the exponential function graphed.

The initial value for the function is not clear in this graph, so we will instead work using two clearer points. There are three clear points: (-1, 1), (1, 2), and (3, 4). As we saw in the last example, two points are sufficient to find the equation for a standard exponential, so we will use the latter two points.
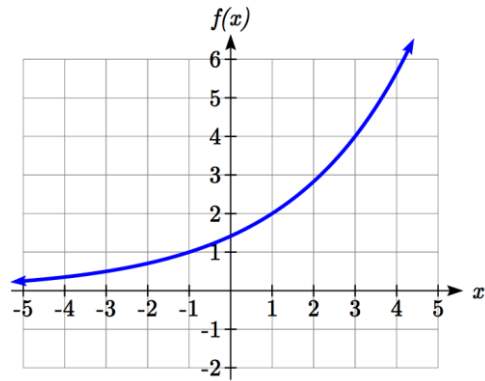
Substituting in (1,2) gives $2 = ab^1$
Substituting in (3,4) gives $4 = ab^3$

Solving the first equation for a gives $a = \dfrac{2}{b}$.

Substituting this expression for a into the second equation:
$4 = ab^3$
$4 = \dfrac{2}{b}b^3 = \dfrac{2b^3}{b}$ Simplify the right-hand side
$4 = 2b^2$
$2 = b^2$
$b = \pm\sqrt{2}$

Since we restrict ourselves to positive values of b, we will use $b = \sqrt{2}$. We can then go back and find a:
$a = \dfrac{2}{b} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

This gives us a final equation of $f(x) = \sqrt{2}(\sqrt{2})^x$.

Compound Interest

In the bank certificate of deposit (CD) example earlier in the section, we encountered compound interest. Typically bank accounts and other savings instruments in which earnings are reinvested, such as mutual funds and retirement accounts, utilize compound interest. The term *compounding* comes from the behavior that interest is earned not on the original value, but on the accumulated value of the account.

In the example from earlier, the interest was compounded monthly, so we took the annual interest rate, usually called the **nominal rate** or **annual percentage rate (APR)** and divided by 12, the number of compounds in a year, to find the monthly interest. The exponent was then measured in months.

Generalizing this, we can form a general formula for compound interest. If the APR is written in decimal form as r, and there are k compounding periods per year, then the interest per compounding period will be r/k. Likewise, if we are interested in the value after t years, then there will be kt compounding periods in that time.

Compound Interest Formula

Compound Interest can be calculated using the formula

$$A(t) = a\left(1 + \frac{r}{k}\right)^{kt}$$

Where

$A(t)$ is the account value

t is measured in years

a is the starting amount of the account, often called the principal

r is the annual percentage rate (APR), also called the nominal rate

k is the number of compounding periods in one year

Example 8

If you invest $3,000 in an investment account paying 3% interest compounded quarterly, how much will the account be worth in 10 years?

Since we are starting with $3000, $a = 3000$
Our interest rate is 3%, so $r = 0.03$
Since we are compounding quarterly, we are compounding 4 times per year, so $k = 4$
We want to know the value of the account in 10 years, so we are looking for $A(10)$, the value when $t = 10$.

$$A(10) = 3000\left(1 + \frac{0.03}{4}\right)^{4(10)} = \$4045.05$$

The account will be worth $4045.05 in 10 years.

<div style="border-left:3px solid #1f3864;padding-left:1em;">

Example 9

A 529 plan is a college savings plan in which a relative can invest money to pay for a child's later college tuition, and the account grows tax free. If Lily wants to set up a 529 account for her new granddaughter, wants the account to grow to $40,000 over 18 years, and she believes the account will earn 6% compounded semi-annually (twice a year), how much will Lily need to invest in the account now?

Since the account is earning 6%, $r = 0.06$
Since interest is compounded twice a year, $k = 2$

In this problem, we don't know how much we are starting with, so we will be solving for a, the initial amount needed. We do know we want the end amount to be $40,000, so we will be looking for the value of a so that $A(18) = 40,000$.

$$40,000 = A(18) = a\left(1 + \frac{0.06}{2}\right)^{2(18)}$$

$$40,000 = a(2.8983)$$

$$a = \frac{40,000}{2.8983} \approx \$13,801$$

Lily will need to invest $13,801 to have $40,000 in 18 years.

</div>

Try it now

5. Recalculate example 2 from above with quarterly compounding.

Because of compounding throughout the year, with compound interest the actual increase in a year is *more* than the annual percentage rate. If $1,000 were invested at 10%, the table below shows the value after 1 year at different compounding frequencies:

Frequency	Value after 1 year
Annually	$1100
Semiannually	$1102.50
Quarterly	$1103.81
Monthly	$1104.71
Daily	$1105.16

If we were to compute the actual percentage increase for the daily compounding, there was an increase of $105.16 from an original amount of $1,000, for a percentage increase of $\frac{105.16}{1000} = 0.10516 = 10.516\%$ increase. This quantity is called the **annual percentage yield (APY)**.

Notice that given any starting amount, the amount after 1 year would be

$A(1) = a\left(1 + \dfrac{r}{k}\right)^k$. To find the total change, we would subtract the original amount, then

to find the percentage change we would divide that by the original amount:

$$\dfrac{a\left(1 + \dfrac{r}{k}\right)^k - a}{a} = \left(1 + \dfrac{r}{k}\right)^k - 1$$

Annual Percentage Yield

The **annual percentage yield** is the actual percent a quantity increases in one year. It can be calculated as

$$APY = \left(1 + \dfrac{r}{k}\right)^k - 1$$

This is equivalent to finding the value of $1 after 1 year, and subtracting the original dollar.

Example 10

Bank A offers an account paying 1.2% compounded quarterly. Bank B offers an account paying 1.1% compounded monthly. Which is offering a better rate?

We can compare these rates using the annual percentage yield – the actual percent increase in a year.

Bank A: $APY = \left(1 + \dfrac{0.012}{4}\right)^4 - 1 = 0.012054 = 1.2054\%$

Bank B: $APY = \left(1 + \dfrac{0.011}{12}\right)^{12} - 1 = 0.011056 = 1.1056\%$

Bank B's monthly compounding is not enough to catch up with Bank A's better APR. Bank A offers a better rate.

A Limit to Compounding

As we saw earlier, the amount we earn increases as we increase the compounding frequency. The table, though, shows that the increase from annual to semi-annual compounding is larger than the increase from monthly to daily compounding. This might lead us to believe that although increasing the frequency of compounding will increase our result, there is an upper limit to this process.

To see this, let us examine the value of $1 invested at 100% interest for 1 year.

Frequency	Value
Annual	$2
Quarterly	$2.441406
Monthly	$2.613035
Daily	$2.714567
Hourly	$2.718127
Once per minute	$2.718279
Once per second	$2.718282

These values do indeed appear to be approaching an upper limit. This value ends up being so important that it gets represented by its own letter, much like how π represents a number.

Euler's Number: e

e is the letter used to represent the value that $\left(1+\dfrac{1}{k}\right)^k$ approaches as k gets big.

$e \approx 2.718282$

Because e is often used as the base of an exponential, most scientific and graphing calculators have a button that can calculate powers of e, usually labeled e^x. Some computer software instead defines a function *exp(x)*, where *exp(x)* = e^x.

Because e arises when the time between compounds becomes very small, e allows us to define **continuous growth** and allows us to define a new toolkit function, $f(x) = e^x$.

Continuous Growth Formula

Continuous Growth can be calculated using the formula
$f(x) = ae^{rx}$
where
a is the starting amount
r is the continuous growth rate

This type of equation is commonly used when describing quantities that change more or less continuously, like chemical reactions, growth of large populations, and radioactive decay.

Example 11

Radon-222 decays at a continuous rate of 17.3% per day. How much will 100mg of Radon-222 decay to in 3 days?

Since we are given a continuous decay rate, we use the continuous growth formula. Since the substance is decaying, we know the growth rate will be negative: $r = -0.173$

$f(3) = 100e^{-0.173(3)} \approx 59.512$ mg of Radon-222 will remain.

Try it Now

6. Interpret the following: $S(t) = 20e^{0.12t}$ if $S(t)$ represents the growth of a substance in grams, and time is measured in days.

Continuous growth is also often applied to compound interest, allowing us to talk about continuous compounding.

Example 12

If $1000 is invested in an account earning 10% compounded continuously, find the value after 1 year.

Here, the continuous growth rate is 10%, so $r = 0.10$. We start with $1000, so $a = 1000$. To find the value after 1 year,

$f(1) = 1000e^{0.10(1)} \approx \$1105.17$

Notice this is a $105.17 increase for the year. As a percent increase, this is

$\frac{105.17}{1000} = 0.10517 = 10.517\%$ increase over the original $1000.

Notice that this value is slightly larger than the amount generated by daily compounding in the table computed earlier.

The continuous growth rate is like the nominal growth rate (or APR) – it reflects the growth rate before compounding takes effect. This is different than the annual growth rate used in the formula $f(x) = a(1+r)^x$, which is like the annual percentage yield – it reflects the *actual* amount the output grows in a year.

While the continuous growth rate in the example above was 10%, the actual annual yield was 10.517%. This means we could write two different looking but equivalent formulas for this account's growth:

$f(t) = 1000e^{0.10t}$ using the 10% continuous growth rate

$f(t) = 1000(1.10517)^t$ using the 10.517% actual annual yield rate.

Important Topics of this Section
Percent growth
Exponential functions
Finding formulas
Interpreting equations
Graphs
Exponential Growth & Decay
Compound interest
Annual Percent Yield
Continuous Growth

Try it Now Answers
1. A & C are exponential functions, they grow by a % not a constant number.

2. B(t) is growing faster ($r = 0.075 > 0.05$), but after 3 years A(t) still has a higher account balance

3. $P(t) = 1000(1 - 0.03)^t = 1000(0.97)^t$
    $P(30) = 1000(0.97)^{30} = 401.0071$

4. $3 = ab^1$, so $a = \dfrac{3}{b}$,

   $4.5 = ab^2$, so $4.5 = \dfrac{3}{b}b^2$. $4.5 = 3b$

   $b = 1.5$. $a = \dfrac{3}{1.5} = 2$

   $f(x) = 2(1.5)^x$

5. 24 months = 2 years. $1000\left(1 + \dfrac{.012}{4}\right)^{4(2)} = \$1024.25$

6. An initial substance weighing 20g is growing at a continuous rate of 12% per day.

Section 4.1 Exercises

For each table below, could the table represent a function that is linear, exponential, or neither?

1.

x	1	2	3	4
$f(x)$	70	40	10	-20

2.

x	1	2	3	4
$g(x)$	40	32	26	22

3.

x	1	2	3	4
$h(x)$	70	49	34.3	24.01

4.

x	1	2	3	4
$k(x)$	90	80	70	60

5.

x	1	2	3	4
$m(x)$	80	61	42.9	25.61

6.

x	1	2	3	4
$n(x)$	90	81	72.9	65.61

7. A population numbers 11,000 organisms initially and grows by 8.5% each year. Write an exponential model for the population.

8. A population is currently 6,000 and has been increasing by 1.2% each day. Write an exponential model for the population.

9. The fox population in a certain region has an annual growth rate of 9 percent per year. It is estimated that the population in the year 2010 was 23,900. Estimate the fox population in the year 2018.

10. The amount of area covered by blackberry bushes in a park has been growing by 12% each year. It is estimated that the area covered in 2009 was 4,500 square feet. Estimate the area that will be covered in 2020.

11. A vehicle purchased for $32,500 depreciates at a constant rate of 5% each year. Determine the approximate value of the vehicle 12 years after purchase.

12. A business purchases $125,000 of office furniture which depreciates at a constant rate of 12% each year. Find the residual value of the furniture 6 years after purchase.

Find a formula for an exponential function passing through the two points.

13. $(0,6), (3,750)$ 14. $(0,3), (2,75)$

15. $(0,2000), (2,20)$ 16. $(0,9000), (3,72)$

17. $\left(-1, \dfrac{3}{2}\right), (3,24)$ 18. $\left(-1, \dfrac{2}{5}\right), (1,10)$

19. $(-2,6), (3,1)$ 20. $(-3,4), (3,2)$

21. $(3,1), (5,4)$ 22. $(2,5), (6,9)$

23. A radioactive substance decays exponentially. A scientist begins with 100 milligrams of a radioactive substance. After 35 hours, 50 mg of the substance remains. How many milligrams will remain after 54 hours?

24. A radioactive substance decays exponentially. A scientist begins with 110 milligrams of a radioactive substance. After 31 hours, 55 mg of the substance remains. How many milligrams will remain after 42 hours?

25. A house was valued at $110,000 in the year 1985. The value appreciated to $145,000 by the year 2005. What was the annual growth rate between 1985 and 2005? Assume that the house value continues to grow by the same percentage. What did the value equal in the year 2010?

26. An investment was valued at $11,000 in the year 1995. The value appreciated to $14,000 by the year 2008. What was the annual growth rate between 1995 and 2008? Assume that the value continues to grow by the same percentage. What did the value equal in the year 2012?

27. A car was valued at $38,000 in the year 2003. The value depreciated to $11,000 by the year 2009. Assume that the car value continues to drop by the same percentage. What was the value in the year 2013?

28. A car was valued at $24,000 in the year 2006. The value depreciated to $20,000 by the year 2009. Assume that the car value continues to drop by the same percentage. What was the value in the year 2014?

29. If $4,000 is invested in a bank account at an interest rate of 7 per cent per year, find the amount in the bank after 9 years if interest is compounded annually, quarterly, monthly, and continuously.

30. If $6,000 is invested in a bank account at an interest rate of 9 per cent per year, find the amount in the bank after 5 years if interest is compounded annually, quarterly, monthly, and continuously.

31. Find the annual percentage yield (APY) for a savings account with annual percentage rate of 3% compounded quarterly.

32. Find the annual percentage yield (APY) for a savings account with annual percentage rate of 5% compounded monthly.

33. A population of bacteria is growing according to the equation $P(t)=1600e^{0.21t}$, with t measured in years. Estimate when the population will exceed 7569.

34. A population of bacteria is growing according to the equation $P(t)=1200e^{0.17t}$, with t measured in years. Estimate when the population will exceed 3443.

35. In 1968, the U.S. minimum wage was $1.60 per hour. In 1976, the minimum wage was $2.30 per hour. Assume the minimum wage grows according to an exponential model $w(t)$, where t represents the time in years after 1960. [UW]
    a. Find a formula for $w(t)$.
    b. What does the model predict for the minimum wage in 1960?
    c. If the minimum wage was $5.15 in 1996, is this above, below or equal to what the model predicts?

36. In 1989, research scientists published a model for predicting the cumulative number of AIDS cases (in thousands) reported in the United States: $a(t)=155\left(\dfrac{t-1980}{10}\right)^{3}$, where t is the year. This paper was considered a "relief", since there was a fear the correct model would be of exponential type. Pick two data points predicted by the research model $a(t)$ to construct a new exponential model $b(t)$ for the number of cumulative AIDS cases. Discuss how the two models differ and explain the use of the word "relief." [UW]

37. You have a chess board as pictured, with squares numbered 1 through 64. You also have a huge change jar with an unlimited number of dimes. On the first square you place one dime. On the second square you stack 2 dimes. Then you continue, always doubling the number from the previous square. [UW]

						63	64
						10	9
1	2	3					8

   a. How many dimes will you have stacked on the 10th square?
   b. How many dimes will you have stacked on the nth square?
   c. How many dimes will you have stacked on the 64th square?
   d. Assuming a dime is 1 mm thick, how high will this last pile be?
   e. The distance from the earth to the sun is approximately 150 million km. Relate the height of the last pile of dimes to this distance.