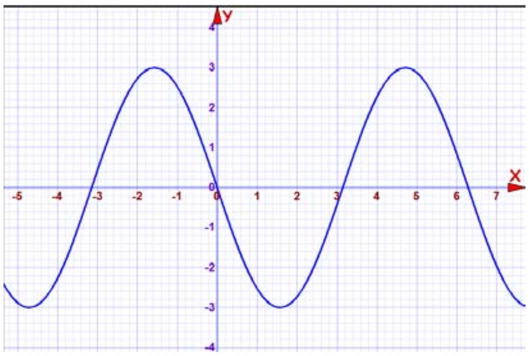
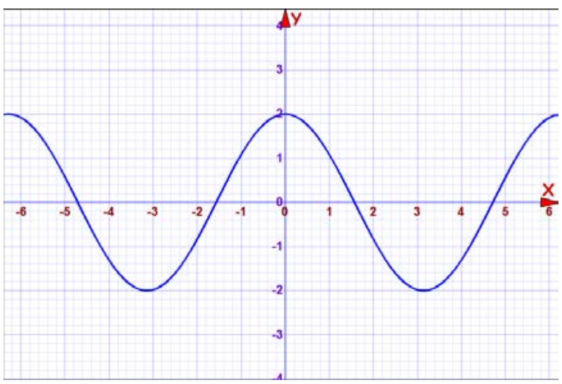
6.1 Solutions to Exercises

1. There is a vertical stretch with a factor of 3, and a horizontal reflection.



3. There is a vertical stretch with a factor of 2.



5. Period: 2. Amplitude: 3. Midline: $y = -4$.

The function is a sine function, because the midline intersects with the y axis, with the form $f(x) = A\sin(Bx) + C$. C is the midline, and A is the amplitude. $B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{2}$, so B is π. So, the formula is $f(x) = 3\sin(\pi x) - 4$.

7. Period: 4π. Amplitude: 2. Midline: $y = 1$.

The function is a cosine function because its maximum intersects with the y axis, which has the form $f(x) = A\cos(Bx) + C$. C is the midline and A is the Amplitude. $B = \frac{2\pi}{\text{Period}}$, so $B = \frac{1}{2}$. The formula is $f(x) = 2\cos\left(\frac{x}{2}\right) + 1$.

9. Period: 5. Amplitude: 2. Midline: $y = 3$.

It is also important to note that it has a vertical reflection, which will make A negative.

The function is a cosine function because its minimum intersects with the y axis, which has the form $f(x) = A\cos(Bx) + C$. C is the midline and A is the Amplitude. $B = \frac{2\pi}{\text{Period}}$, so $B = \frac{2\pi}{5}$. The formula is $f(x) = -2\cos\left(\frac{2\pi}{5}x\right) + 3$.

11. Amplitude: 3. The period is $\frac{2\pi}{B}$, where B is the coefficient in front of x. Period $= \frac{\pi}{4}$. Horizontal shift: 4 to the left. Midline: $y = 5$.

13. Amplitude: 2. The period is $\frac{2\pi}{B}$, where B is the coefficient in front of x. Period $= \frac{2\pi}{3}$. In order to find the horizontal shift the inside of the sine operator must be factored. $3x - 21 = 3(x - 7)$. Horizontal shift: 7 to the right. Midline: $y = 4$.

15. Amplitude: 1. The period is $\frac{2\pi}{B}$, where B is the coefficient in front of x. Period: 12. In order to find the horizontal shift, the inside of the sine operator must be factored. $\frac{\pi}{6}x + \pi = \frac{\pi}{6}(x + 6)$. Horizontal shift: 6 to the left. Midline: $y = -3$.

17. To find the formula of the function we must find the Amplitude, stretch or shrink factor, horizontal shift, and midline.

The graph oscillates from a minimum of -4 to a maximum of 4, so the midline is at $y = 0$ because that is halfway between.

The Amplitude is the distance between the midline and the maximum or minimum, so the Amplitude is 4.

The stretch or shrink factor is $\frac{2\pi}{\text{Period}}$. The period is 10 which is the distance from one peak to the next peak. So, $B = \frac{2\pi}{10}$ or $\frac{\pi}{5}$.

To find the horizontal shift we must first decide whether to use sine or cosine. If you were to use cosine the horizontal shift would be $\frac{3}{2}$ to the right. If you were to use sine it would be 1 to the left. Thus the formula could be $f(x) = 4\sin\left(\frac{\pi}{5}(x + 1)\right)$ or $f(x) = 4\cos\left(\frac{\pi}{5}\left(x - \frac{3}{2}\right)\right)$.

19. To find the formula of the function we must find the Amplitude, stretch or shrink factor, horizontal shift, and midline.

The graph oscillates from a minimum of -1 to a maximum of 1, so the midline is at $y = 0$ because that is halfway between.

The Amplitude is the distance between the midline and the maximum or minimum, so the Amplitude is 1.

The stretch or shrink factor is $\frac{2\pi}{\text{Period}}$. The period is 10 which is the distance from one peak to the next peak. So, $B = \frac{2\pi}{10}$ or $\frac{\pi}{5}$.

To find the horizontal shift we must first decide whether to use sine or cosine. If you were to use cosine the horizontal shift would be 2 to the left. If you were to use sine it would be $\frac{1}{2}$ to the right, but the function would need a vertical reflection since the minimum is to the right of the y axis rather than the maximum.

Thus the formula could be $f(x) = -\sin\left(\frac{\pi}{5}\left(x - \frac{1}{2}\right)\right)$ or $f(x) = \cos\left(\frac{\pi}{5}(x + 2)\right)$.

21. To find the formula of the function we must find the Amplitude, stretch or shrink factor, horizontal shift, and midline.

Since the maximum temperature is 57 and the minimum is 43 degrees, the midline is 50 since that is halfway in between.

The amplitude is 7 because that is the difference between the midline and either the max or the min.

Since the temperature at $t = 0$ is 50 degrees, sine is the best choice because the midline intersects with vertical axis.

The function must have a vertical reflection because the lowest temperature generally happens in the morning rather than in the afternoon. Having a negative sine function will put the minimum in the morning and the maximum in the afternoon.

Our independent variable t is in hours so the period is 24 because there are 24 hours in the day.

So, our function is $D(t) = 50 - 7\sin\left(\frac{2\pi}{24}t\right)$ which is $D(t) = 50 - 7\sin\left(\frac{\pi}{12}t\right)$.

23. a. The period is 10 minutes because that is how long it takes to get from one point on the Ferris wheel to that same point again.

The maximum point on the Ferris wheel is 26 meters, because it is the height of the wheel plus the extra 1 meter that it is off the ground. The minimum point is 1 m since the wheel is 1 meter off the ground. The Amplitude is half of the distance between the maximum and the minimum which is 12.5 meters.

The Midline is where the center of the Ferris wheel is which is one meter more than the Amplitude, because the Ferris wheel starts 1 m off the ground. The Midline is $y = 13.5$ meters.

b. The formula is $h(t) = -12.5\cos\left(\frac{2\pi}{10}t\right)+13.5$. The function is a negative cosine function because the Ferris wheel starts at the minimum height.

c. Plug 5 minutes in for t in the height formula: $h(5) = -12.5\cos\left(\frac{2\pi}{10}\cdot 5\right) + 13.5 = 26$ meters.

6.2 Solutions to Exercises

1. Features of the graph of $f(x) = \tan(x)$ include:

- The period of the tangent function is π.
- The domain of the tangent function is $\theta \neq \frac{\pi}{2} + k\pi$, where k is an integer.

- The range of the tangent function is all real numbers, (- ∞, ∞).

Therefore the matching graph for $f(x) = \tan(x)$ is II.

3. Features of the graph of $f(x) = \csc(x)$ include:

- The period of the cosecant function is 2π.
- The domain of the cosecant function is $\theta \neq k\pi$, where *k* is an integer.
- The range of the cosecant function is (- ∞, -1] ∪[1, ∞).

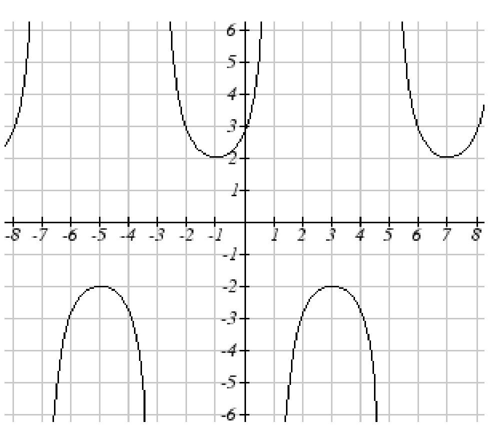
Therefore the matching graph for $f(x) = \csc(x)$ is I.

5. Since the period of a tangent function is π, the period of $f(x)$ is $\frac{\pi}{4}$, and the horizontal shift is 8 units to the right.

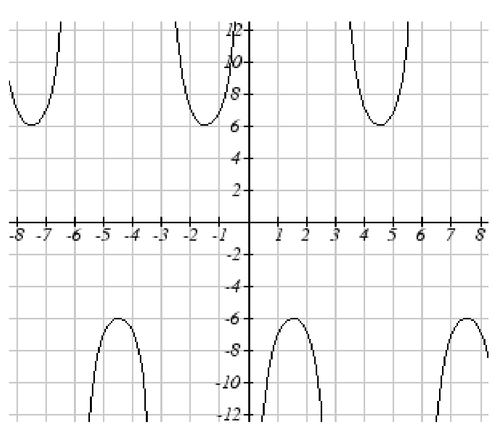
7. Since the period of a secant function is 2π, the period of $h(x)$ is $\frac{2\pi}{\frac{\pi}{4}}$ = 8, and the horizontal shift is 1 unit to the left.

9. Since the period of a cosecant function is 2π, the period of $m(x)$ is $\frac{2\pi}{\frac{\pi}{3}}$ = 6, and the horizontal shift is 3 units to the left.

11. A graph of $h(x) = 2\sec(\frac{\pi}{4}(x+1))$



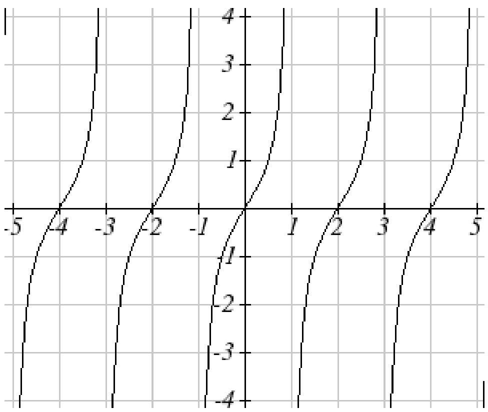
13. A graph of $m(x) = 6\csc(\frac{\pi}{3}x + \pi)$



15. To graph $j(x) = \tan(\frac{\pi}{2}\ x)$:

The period of $j(x)$ is $\pi/\frac{\pi}{2} = 2$, so the horizontal stretch should be 2 units in length. Since the coefficient of $j(x)$ is 1, there is no vertical stretch.

The period of a tangent function is π, and its domain is $\theta \neq \frac{\pi}{2} + k\pi$, where *k* is an integer.



17. The graph of $f(x)$ has the shape of either a secant or a cosecant function. However, since it has domain of $x \neq \frac{\pi}{2} + k\pi$ (*k* is an integer), it should be the graph of a secant function.

Assume that the function has form of f(x) = a sec(kx) +b.

- Because period of the graph is 2, it is compressed by $\frac{2}{\pi}$ or k = $\frac{\pi}{2}$.
- The graph is shifted down by 1 unit, then b = -1.
- To find a, use the point (0,1). Substitute this into the formula of f(x), we have

$$1 = a\sec(0) - 1$$

$$1 = a - 1$$

$$a = 2$$

Thus, a formula of the function graphed above is $f(x) = 2\sec(\frac{\pi}{2}x) - 1$.

19. The graph of $h(x)$ has the shape of either a secant or a cosecant function. However, since it has domain of $x \neq k\pi$ (*k* is an integer), it should be the graph of a cosecant function.

Assume that the function has form of $h(x) = a\csc(kx) + b$.

- Because period of the graph is 4, it is compressed by $\frac{4}{\pi}$ or k = $\frac{\pi}{4}$.
- The graph is shifted up by 1 unit, then b = 1.
- To find a, use the point (2,3). Substitute this into the formula of h(x), we have

$$3 = a\csc\left(\frac{\pi}{4}\cdot 2\right) + 1$$

$$3 = a + 1$$

$$a = 2$$

Thus, a formula of the function graphed above is $h(x) = 2\csc(\frac{\pi}{4}x) + 1$.

21. tan (-x) = -tan x = - (-1.5) = 1.5

23. sec (-x) = -sec x = -2

25. csc (-x) = -csc x = -(-5) = 5

27. cot (-x) cos (-x) + sin (-x) = (-cot x)(cos x) – sin x = (- $\frac{\cos x}{\sin x}$)(cos x) – sin x

$$= -\frac{\cos^2 x}{\sin x} - \sin x = \frac{-(\cos^2 x + \sin^2 x)}{\sin x} = -\frac{1}{\sin x} = -\csc x.$$

6.3 Solutions to Exercises

1. For $\sin^{-1}(\frac{\sqrt{2}}{2})$, we are looking for an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with a sine value of $\frac{\sqrt{2}}{2}$. The angle that satisfies this is $\sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$.

3. For $\sin^{-1}(-\frac{1}{2})$, we are looking for an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with a sine value of $-\frac{1}{2}$. The angle that satisfies this is $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$.

5. For $\cos^{-1}(\frac{1}{2})$, we are looking for an angle in $[0,\pi]$ with a cosine value of $\frac{1}{2}$. The angle that satisfies this is $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$.

7. For $\cos^{-1}(-\frac{\sqrt{2}}{2})$, we are looking for an angle in $[0,\pi]$ with a cosine value of $-\frac{\sqrt{2}}{2}$. The angle that satisfies this is $\cos^{-1}(-\frac{\sqrt{2}}{2}) = \frac{3\pi}{4}$.

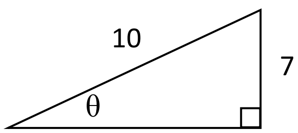
9. For $\tan^{-1}(1)$, we are looking for an angle in $(-\frac{\pi}{2},\frac{\pi}{2})$ with a tangent value of 1. The angle that satisfies this is $\tan^{-1}(1) = \frac{\pi}{4}$.

11. For $\tan^{-1}(-\sqrt{3})$, we are looking for an angle in $(-\frac{\pi}{2},\frac{\pi}{2})$ with a tangent value of $-\sqrt{3}$. The angle that satisfies this is $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$.

13. In radian mode, $\cos^{-1}(-0.4) \approx 1.9823$ rad or 1.9823. In degree mode, $\cos^{-1}(-0.4) \approx 113.5782°$.

15. In radian mode, $\sin^{-1}(-0.8) \approx -0.9273$ rad or -0.9273. In degree mode, $\sin^{-1}(-0.8) \approx -53.131°$.

17.



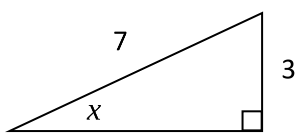
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{7}{10} = 0.7$$

So $\theta = \sin^{-1}(0.7) \approx 0.7754$.

19. $\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$. For $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$, we are looking for an angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a sine value of $\frac{\sqrt{2}}{2}$. The angle that satisfies this is $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$. So $\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$.

21. $\sin^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) = \sin^{-1}\left(-\frac{1}{2}\right)$. For $\sin^{-1}\left(-\frac{1}{2}\right)$, we are looking for an angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a sine value of $-\frac{1}{2}$. The angle that satisfies this is $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$. So $\sin^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.

23. Let $x = \sin^{-1}\left(\frac{3}{7}\right)$. Then $\sin(x) = \frac{3}{7} = \frac{\text{opposite}}{\text{hypotenuse}}$.



Using the Pythagorean Theorem, we can find the adjacent of the triangle:

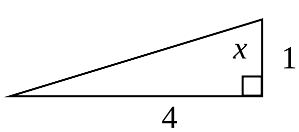
$3^2 + \text{adjacent}^2 = 7^2$

$\text{adjacent}^2 = 7^2 - 3^2 = 49 - 9 = 40$

$\text{adjacent} = \sqrt{40} = 2\sqrt{10}$

Therefore, $\cos\left(\sin^{-1}\left(\frac{3}{7}\right)\right) = \cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{10}}{7}$.

25. Let $x = \tan^{-1}(4)$, then $\tan(x) = 4 = \frac{\text{opposite}}{\text{adjacent}}$.



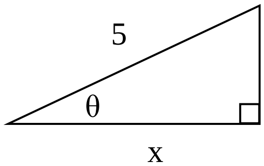
Using the Pythagorean Theorem, we can find the hypotenuse of the triangle:

$\text{hypotenuse}^2 = 1^2 + 4^2 = 1 + 16 = 17$

$\text{hypotenuse} = \sqrt{17}$

Therefore, $\cos(\tan^{-1}(4)) = \cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{17}}$.

27. Let $\theta = \cos^{-1}\left(\frac{x}{5}\right)$, then $\cos(\theta) = \frac{x}{5} = \frac{\text{adjacent}}{\text{hypotenuse}}$



Using the Pythagorean Theorem, we can find the opposite of the triangle:

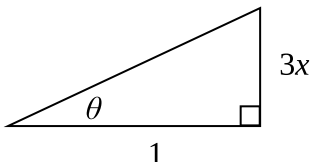
$x^2 + \text{opposite}^2 = 5^2 = 25$

$\text{opposite}^2 = 25 - x^2$

$\text{opposite} = \sqrt{25 - x^2}$

Therefore, $\sin\left(\cos^{-1}\left(\frac{x}{5}\right)\right) = \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{25 - x^2}}{5}$.

29. Let $\theta = \tan^{-1}(3x)$, then $\tan(\theta) = 3x = \frac{\text{opposite}}{\text{adjacent}}$



Using the Pythagorean Theorem, we can find the hypotenuse of the triangle:

hypotenuse^2 = $(3x)^2 + 1^2 = 9x^2 + 1$

hypotenuse = $\sqrt{9x^2 + 1}$

Therefore, $\sin(\tan^{-1}(3x)) = \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3x}{\sqrt{9x^2+1}}$.

6.4 Solutions to Exercises

1. $2\sin(\theta) = -\sqrt{2}$

   $\sin(\theta) = \frac{-\sqrt{2}}{2}$

   $\theta = \frac{5\pi}{4} + 2k\pi$ or $\theta = \pi - \frac{\pi}{4} + 2k\pi = \frac{7\pi}{4} + 2k\pi$, for $k \in Z$.

Since $0 \le \theta < 2\pi$, the answers should be $\theta = \frac{5\pi}{4}$ and $\theta = \frac{7\pi}{4}$.

3. $2\cos(\theta) = 1$

   $\cos(\theta) = \frac{1}{2}$

   $\theta = \frac{\pi}{3} + 2k\pi$ or $\theta = 2\pi - \frac{\pi}{3} + 2k\pi = \frac{5\pi}{3} + 2k\pi$, for $k \in Z$.

Since $0 \le \theta < 2\pi$, the answers should be $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

5. $\sin(\theta) = 1$

   $\theta = \frac{\pi}{2} + 2k\pi$, for $k \in Z$.

Since $0 \le \theta < 2\pi$, the answer should be $\theta = \frac{\pi}{2}$.

7. $\cos(\theta) = 0$

   $\theta = \frac{\pi}{2} + 2k\pi$ or $\theta = 2\pi - \frac{\pi}{2} + 2k\pi = \frac{3\pi}{2} + 2k\pi$, for $k \in Z$.

Since $0 \le \theta < 2\pi$, the answers should be $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$.

9. $2\cos(\theta) = \sqrt{2}$

   $\cos(\theta) = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} + 2k\pi$ or $\theta = 2\pi - \frac{\pi}{4} + 2k\pi = \frac{7\pi}{4} + 2k\pi$, for $k \in Z$.

11. $2 \sin(\theta) = -1$

$\sin(\theta) = -\frac{1}{2}$

$\theta = \frac{7\pi}{6} + 2k\pi$ or $\theta = \frac{11\pi}{6} + 2k\pi$, for $k \in Z$.

13. $2 \sin(3\theta) = 1$

$\sin(3\theta) = \frac{1}{2}$

$3\theta = \frac{\pi}{6} + 2k\pi$ or $3\theta = \frac{5\pi}{6} + 2k\pi$, for $k \in Z$

$\theta = \frac{\pi}{18} + \frac{2k\pi}{3}$ or $\theta = \frac{5\pi}{18} + \frac{2k\pi}{3}$, for $k \in Z$.

15. $2 \sin(3\theta) = -\sqrt{2}$

$\sin(3\theta) = -\frac{\sqrt{2}}{2}$

$3\theta = \frac{5\pi}{4} + 2k\pi$ or $3\theta = \frac{7\pi}{4} + 2k\pi$, for $k \in Z$

$\theta = \frac{5\pi}{12} + \frac{2k\pi}{3}$ or $\theta = \frac{7\pi}{12} + \frac{2k\pi}{3}$, for $k \in Z$.

17. $2 \cos(2\theta) = 1$

$\cos(2\theta) = \frac{1}{2}$

$2\theta = \frac{\pi}{3} + 2k\pi$ or $2\theta = \frac{5\pi}{3} + 2k\pi$, for $k \in Z$

$\theta = \frac{\pi}{6} + k\pi$ or $\theta = \frac{5\pi}{6} + k\pi$, for $k \in Z$.

19. $2 \cos(3\theta) = -\sqrt{2}$

$\cos(3\theta) = -\frac{\sqrt{2}}{2}$

$3\theta = \frac{3\pi}{4} + 2k\pi$ or $3\theta = \frac{5\pi}{4} + 2k\pi$, for $k \in Z$

$\theta = \frac{\pi}{4} + \frac{2k\pi}{3}$ or $\theta = \frac{5\pi}{12} + \frac{2k\pi}{3}$, for $k \in Z$.

21. $\cos\left(\frac{\pi}{4}\theta\right) = -1$

$\frac{\pi}{4}\theta = \pi + 2k\pi$, for $k \in Z$

$\theta = 4 + 8k$, for $k \in Z$.

23. $2\sin(\pi\theta) = 1$

$\sin(\pi\theta) = \frac{1}{2}$

$\pi\theta = \frac{\pi}{6} + 2k\pi$ or $\pi\theta = \frac{5\pi}{6} + 2k\pi$, for $k \in Z$

$\theta = \frac{1}{6} + 2k$ or $\theta = \frac{5}{6} + 2k$, for $k \in Z$.

25. $\sin(x) = 0.27$

$x = \sin^{-1}(0.27)$

$x = 0.2734 + 2k\pi$ or $x = \pi - 0.2734 + 2k\pi$, for $k \in Z$.

Since $0 \le x < 2\pi$, there are two answers, $x = 0.2734$ and $x = \pi - 0.2734 = 2.8682$.

27. $\sin(x) = -0.58$

$x = \sin^{-1}(-0.58)$

$x = -0.6187 + 2k\pi$ or $x = \pi - (-0.6187) + 2k\pi$, for $k \in Z$.

Since $0 \le x < 2\pi$, there are two answers, $x = -0.6187 + 2\pi = 5.6645$ and $x = \pi - (-0.61873) = 3.7603$.

29. $\cos(x) = -0.55$

$x = \cos^{-1}(-0.55)$

$x = 2.1532 + 2k\pi$ or $x = 2\pi - 2.1532 + 2k\pi$, for $k \in Z$.

Since $0 \le x < 2\pi$, there are two answers, $x = 2.1532$ and $x = 2\pi - 2.1532 = 4.13$.

31. $\cos(x) = 0.71$

$x = \cos^{-1}(0.71)$

$x = 0.7813 + 2k\pi$ or $x = 2\pi - 0.7813 + 2k\pi$, for $k \in Z$.

Since $0 \le x < 2\pi$, there are two answers, $x = 0.7813$ or $x = 2\pi - 0.7813 = 5.5019$.

33. $7\sin(6x) = 2$

$\sin(6x) = \frac{2}{7}$

$6x = \sin^{-1}(\frac{2}{7})$

$6x = 0.28975 + 2k\pi$ or $6x = \pi - 0.28975 + 2k\pi$, for $k \in Z$.

In order to get the first two positive solutions, k = 0. This leads to:

$6x = 0.28975$ or $6x = \pi - 0.28975 = 2.85184$

$x = 0.04829$ or $x = 0.47531$

35. $5\cos(3x) = -3$

$\cos(3x) = -\frac{3}{5}$

$3x = \cos^{-1}(-\frac{3}{5})$

$3x = 2.2143 + 2k\pi$ or $3x = 2\pi - 2.2143 + 2k\pi$, for $k \in Z$.

In order to get the first two positive solutions, k = 0. This leads to:

$3x = 2.2143$ or $3x = 2\pi - 2.2143 = 4.0689$

$x = 0.7381$ or $x = 1.3563$

37. $3\sin(\frac{\pi}{4}x) = 2$

$\sin(\frac{\pi}{4}x) = \frac{2}{3}$

$\frac{\pi}{4}x = \sin^{-1}(\frac{2}{3})$

$\frac{\pi}{4}x = 0.72973 + 2k\pi$ or $\frac{\pi}{4}x = \pi - 0.72973 + 2k\pi$, for $k \in Z$.

In order to get the first two positive solutions, k = 0. This leads to:

$\frac{\pi}{4}x = 0.72973$ or $\frac{\pi}{4}x = \pi - 0.72973 = 2.41186$

$x = 0.9291$ or $x = 3.0709$

39. $5\cos(\frac{\pi}{3}x) = 1$

$\cos(\frac{\pi}{3}x) = \frac{1}{5}$

$\frac{\pi}{3}x = \cos^{-1}(\frac{1}{5})$

$\frac{\pi}{3}x = 1.3694 + 2k\pi$ or $\frac{\pi}{3}x = 2\pi - 1.3694 + 2k\pi$, for $k \in Z$.

In order to get the first two positive solutions, k = 0. This leads to:

$\frac{\pi}{3}x = 1.3694$ or $\frac{\pi}{3}x = 2\pi - 1.3694 = 4.9138$

$x = 1.3077$ or $x = 4.6923$

6.5 Solutions to Exercises

1. $c = \sqrt{5^2 + 8^2} = \sqrt{89}$

   $A = \tan^{(-1)}(8/5) \approx 57.9946°$

   $B = \tan^{(-1)}(5/8) \approx 32.0054°$

3. $b^2 = 15^2 - 7^2$, $b = \sqrt{225 - 49} = \sqrt{176}$

   $A = \sin^{(-1)}(7/15) \approx 27.8181°$

   $B = \cos^{(-1)}(7/15) \approx 62.1819°$

5. Note that the function has a maximum of 10 and a minimum of -2. The function returns to its maximum or minimum every 4 units in the x direction, so the period is 4.

Midline = 4 because 4 lies equidistant from the function's maximum and minimum.

Amplitude = 10 – 4 or 4 – (-2) = 6

Horizontal compression factor = $\frac{2\pi}{period} = \frac{2\pi}{4} = \frac{\pi}{2}$

If we choose to model this function with a sine curve, then a horizontal shift is required. Sin(x) will begin its period at the midline, but our function first reaches its midline at $x = 1$. To adjust for this, we can apply a horizontal shift of -1.

Therefore, we may model this function with $y(x) = 6\sin\left(\frac{\pi}{2}(x-1)\right) + 4$

7. We are given when the minimum temperature first occurs, so it would be a good choice to create our model based on a flipped cosine curve with a period of 24 hours.

Amplitude = $\frac{63-37}{2} = 13$

Midline = $\frac{63+37}{2} = 50$

Horizontal stretch factor $= \frac{2\pi}{24} = \frac{\pi}{12}$

Horizontal shift = -5

Using this information we have the following model: $D(t) = -13\cos\left(\frac{\pi}{12}(t-5)\right) + 50$

9. a. We are given when the population is at a minimum, so we can create a model using a flipped cosine curve with a period of 12 months.

Midline = 129

Amplitude = 25

Horizontal stretch factor = $\frac{2\pi}{12} = \frac{\pi}{6}$

Using this information we have the following model: $P(t) = -25\cos\left(\frac{\pi}{6}(t)\right) + 129$

b. April is 3 months after January which means that we may use the previous model with a rightward shift of 3 months. $P(t) = -25\cos\left(\frac{\pi}{6}(t-3)\right) + 129$

11. Let $D(t)$ be the temperature in farenheight at time *t*, where *t* is measured in hours since midnight. We know when the maximum temperature occurs so we can create a model using a cosine curve.

Midline = 85

Amplitude = 105 - 85 = 20

Horizontal stretch factor = $\frac{2\pi}{24} = \frac{\pi}{12}$

Horizontal shift = -17

Using this information we have the following model: $D(t) = 20\cos\left(\frac{\pi}{12}(t-17)\right) + 85$

$$D(9) = 20\cos\left(\frac{\pi}{12}(9-17)\right) + 85 = 75$$

Therefore the temperature at 9 AM was 75° F.

13. Let D(t) be the temperature in farenheight at time t, where t is measured in hours since midnight. We know when the average temperature first occurs so we can create a model using a sine curve. The average temperature occurs at 10 AM we can assume that the temperature increases after that.

Midline = $\frac{63+47}{2} = 55$

Amplitude = $\frac{63-47}{2} = 8$

Horizontal stretch factor = $\frac{2\pi}{24} = \frac{\pi}{12}$

Horizontal shift = -10

$$D(t) = 8\sin\left(\frac{\pi}{12}(t-10)\right) + 55$$

To calculate when the temperature will first be 51°F we set $D(t)$ = 51 and solve for t.

$$51 = 8\sin\left(\frac{\pi}{12}(t-10)\right) + 55$$

$$\sin\left(\frac{\pi}{12}(t-10)\right) = -\frac{1}{2}$$

$$\frac{\pi}{12}(t-10) = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$t = \frac{-\pi/6}{\pi/12} + 10 = 8$$

The first time the temperature reaches 51°F for the day is at 8 AM, 8 hours past midnight.

15. Let $h(t)$ be the height from the ground in meters of your seat on the ferris wheel at a time *t*, where *t* is measured in minutes. The minimum is level with the platform at 2 meters, and the maximum is the 2 meter platform plus the 20 meter diameter. Since you begin the ride at a minimum, a flipped cosine function would be ideal for modeling this situation with a period being a full revolution.

Midline = $\frac{22+2}{2} = 12$

Amplitude = $\frac{22-2}{2} = 10$

Horizontal compression factor= $\frac{2\pi}{6} = \frac{\pi}{3}$

Putting this information gives us $h(t) = -10\cos\left(\frac{\pi}{3}(t)\right) + 12$

To find the amount of time which the height is above 13 meters, we can set $h(t)$ = 13 and find both values of *t* for which this is true and take their difference.

$$13 = -10\cos\left(\frac{\pi}{3}(t)\right) + 12$$

$$\cos\left(\frac{\pi}{3}(t)\right) = -\frac{1}{10}$$

$\frac{\pi}{3}t_1 = \cos^{-1}\left(-\frac{1}{10}\right)$ and $\frac{\pi}{3}t_2 = 2\pi - \cos^{-1}\left(-\frac{1}{10}\right)$

$$t_1 - t_2 = \frac{3}{\pi}(2\pi - \cos^{-1}\left(-\frac{1}{10}\right) - \cos^{-1}\left(-\frac{1}{10}\right) \approx 2.80869431742$$

Therefore you would be 13 meters or more above the ground for about 2.8 minutes during the ride.

17. Let $S(t)$ be the amount of sea ice around the north pole in millions of square meters at time *t*, where *t* is the number of months since January. Since we know when the maximum occurs, a cosine curve would be useful in modeling this situation.

Midline = $\frac{6+14}{2} = 10$

Amplitude = $\frac{6-14}{2} = 4$

Horizontal stretch factor = $\frac{2\pi}{12} = \frac{\pi}{6}$

Horizontal shift = -2

Therefore $S(t) = 4\cos\left(\frac{\pi}{6}(t-2)\right) + 10$

To find where there will be less than 9 million square meters of sea ice we need to set $S(t)$= 9 and find the difference between the two *t* values in which this is true during a single period.

$$4\cos\left(\frac{\pi}{6}(t-2)\right) + 10 = 9$$

$$\cos\left(\frac{\pi}{6}(t-2)\right) = -\frac{1}{4}$$

$\frac{\pi}{6}(t_1 - 2) = \cos^{-1}\left(-\frac{1}{4}\right)$ and $\frac{\pi}{6}(t_2 - 2) = 2\pi - \cos^{-1}\left(-\frac{1}{4}\right)$

Solving these equations gives:

$$t_1 = \frac{6}{\pi}\left(\cos^{-1}\left(-\frac{1}{4}\right)\right) + 2 \approx 5.4825837$$

$$t_2 = \frac{6}{\pi}\left(2\pi - \cos^{-1}\left(-\frac{1}{4}\right)\right) + 2 \approx 10.5174162$$

$$t_1 - t_2 \approx 5.0348325$$

Therefore there are approximately 5.035 months where there is less than 9 million square meters of sea ice around the north pole in a year.

19. a. We are given when the smallest breath occurs, so we can model this using a flipped cosine function. We're not explicitly told the period of the function, but we're given when the largest and smallest breath occurs. It takes half of cosine's period to go from the smallest to largest value. We can find the period by doubling the difference of the *t* values that correspond to the largest and smallest values.

Period = 2(55 – 5) = 100

Midline = $\frac{0.6 + 1.8}{2} = 1.2$

Amplitude = $\frac{1.8 - 0.6}{2} = 0.6$

Horizontal shift = -5

Horizontal stretch factor = $\frac{2\pi}{100} = \frac{\pi}{50}$

This gives us $b(t) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(t-5)\right)$.

b. $b(5) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(5-5)\right) = 0.6$

$b(10) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(10-5)\right) \approx 0.63$

$b(15) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(15-5)\right) \approx 0.71$

$b(20) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(20-5)\right) \approx 0.85$

$b(25) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(25-5)\right) \approx 1.01$

$b(30) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(30-5)\right) = 1.2$

$b(35) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(35-5)\right) \approx 1.39$

$$b(40) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(40-5)\right) \approx 1.55$$

$$b(45) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(45-5)\right) \approx 1.69$$

$$b(50) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(50-5)\right) \approx 1.77$$

$$b(55) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(55-5)\right) = 1.8$$

$$b(60) = 1.2 - 0.6\cos\left(\frac{\pi}{50}(60-5)\right) \approx 1.77$$

21. a. From either right triangle in the image, $\sin(\alpha) = \frac{3960}{3960+t}$, so $\alpha(t) = \sin^{-1}\left(\frac{3960}{3960+t}\right)$.

b. $\alpha(30{,}000) = \sin^{-1}\left(\frac{3960}{3960+30000}\right)$. The angle opposite of alpha is $90° - \alpha \approx 83.304°$. Twice this angle would be 166.608°, which is the angle of one end of the satellite's coverage on earth to the other end. The ratio of this angle to the 360° angle needed to cover the entire earth is $\frac{166.608°}{360°} = .4628$ or 46.28%. Therefore it would take 3 satellites to cover the entire circumference of the earth.

c. Using the same methods as in 21b, we find $\alpha = 52.976°$. The angle between one end of the satellite's coverage to the other is $2(90° - 52.976°) = 74.047°$. The ratio of this angle to the 360° angle needed to cover the entire earth is $\frac{74.047°}{360°} \approx 0.206$. Therefore roughly 20.6% of the earth's circumference can be covered by one satellite. This means that you would need 5 satellites to cover the earth's circumference.

d. Using the same methods as in 21b and 21c we can solve for *t*. $\alpha = \sin^{-1}\left(\frac{3960}{3960+t}\right)$. So the angle from one end of the satellite's coverage to the other is $2\left(90° - \sin^{-1}\left(\frac{3960}{3960+t}\right)\right)$. The ratio of this angle to the 360° angle needed to cover the earth is 0.2, so we have:

$$\frac{2\left(90° - \sin^{-1}\left(\frac{3960}{3960+t}\right)\right)}{360°} = 0.2$$

$$90° - \sin^{-1}\left(\frac{3960}{3960+t}\right) = 36°$$

$$\sin^{-1}\left(\frac{3960}{3960+t}\right) = 54°$$

$$\frac{3960}{3960+t} = \sin(54°)$$

$$3960\sin(54°) + t\sin(54°) = 3960$$

$$t = 3960\ \frac{1-\sin(54°)}{\sin(54°)} \approx 934.829$$

To cover 20% of the earth's circumference, a satellite would need to be placed approximately 934.829 miles from the earth's surface.