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Precalculus: An Investigation of Functions

Student Solutions Manual for Chapter 5

5.1 Solutions to Exercises

1. $D = \sqrt{(5-(-1))^2+(3-(-5))^2} = \sqrt{(5+1)^2+(3+5)^2} = \sqrt{6^2+8^2} = \sqrt{100} = 10$

3. Use the general equation for a circle:
$(x-h)^2+(y-k)^2=r^2$

We set $h=8$, $k=-10$, and $r=8$:
$(x-8)^2+(y-(-10))^2=8^2$
$(x-8)^2+(y+10)^2=64$

5. Since the circle is centered at (7, -2), we know our equation looks like this:
$(x-7)^2+\left(y-(-2)\right)^2=r^2$
$(x-7)^2+(y+2)^2=r^2$

What we don't know is the value of r, which is the radius of the circle. However, since the circle passes through the point (-10, 0), we can set $x=-10$ and $y=0$:
$((-10)-7)^2+(0+2)^2=r^2$
$(-17)^2+2^2=r^2$

Flipping this equation around, we get:
$r^2=289+4$
$r^2=293$

Note that we actually don't need the value of r; we're only interested in the value of r^2. Our final equation is:
$(x-7)^2+(y+2)^2=293$

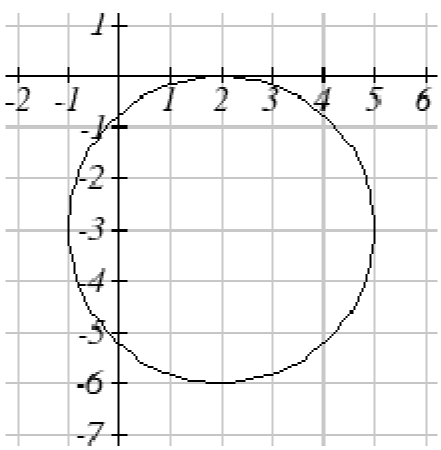
7. If the two given points are endpoints of a diameter, we can find the length of the diameter using the distance formula:

$d = \sqrt{(8-2)^2 + (10-6)^2} = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$

Our radius, r, is half this, so $r = \sqrt{13}$ and $r^2 = 13$. We now need the center (h, k) of our circle. The center must lie exactly halfway between the two given points: $h = \frac{8+2}{2} = \frac{10}{2} = 5$ and $k = \frac{10+6}{2} = \frac{16}{2} = 8$. So:

$(x-5)^2 + (y-8)^2 = 13$

9. This is a circle with center (2, -3) and radius 3:



11. The equation of the circle is:

$(x-2)^2 + (y-3)^2 = 3^2$

The circle intersects the y-axis when $x = 0$:

$(0-2)^2 + (y-3)^2 = 3^2$

$(-2)^2 + (y-3)^2 = 9$

$4 + (y-3)^2 = 9$

$(y-3)^2 = 5$

$y - 3 = \pm\sqrt{5}$

The y intercepts are $(0, 3+\sqrt{5})$ and $(0, 3-\sqrt{5})$.

13. The equation of the circle is:
$(x-0)^2+(y-5)^2=3^2$, so $x^2+(y-5)^2=9$.

The line intersects the circle when $y=2x+5$, so substituting for y:
$x^2+((2x+5)-5)^2=9$
$x^2+(2x)^2=9$
$5x^2=9$
$x^2={}^9/_5$
$x=\pm\sqrt{{}^9/_5}$

Since the question asks about the intersection in the first quadrant, x must be positive.
Substituting $x=\sqrt{{}^9/_5}$ into the linear equation $y=2x+5$, we find the intersection at $\left(\sqrt{{}^9/_5}, 2\sqrt{{}^9/_5}+5\right)$ or approximately (1.34164, 7.68328). (We could have also substituted $x=\sqrt{{}^9/_5}$ into the original equation for the circle, but that's more work.)

15. The equation of the circle is:
$(x-(-2))^2+(y-0)^2=3^2$, so $(x+2)^2+y^2=9$.

The line intersects the circle when $y=2x+5$, so substituting for y:
$(x+2)^2+(2x+5)^2=9$
$x^2+4x+4+4x^2+20x+25=9$
$5x^2+24x+20=0$

This quadratic formula gives us $x\approx-3.7266$ and $x\approx-1.0734$. Plugging these into the linear equation gives us the two points (-3.7266, -2.4533) and (-1.0734, 2.8533), of which only the second is in the second quadrant. The solution is therefore (-1.0734, 2.8533).

17. Place the transmitter at the origin (0, 0). The equation for its transmission radius is then:
$x^2+y^2=53^2$

Your driving path can be represented by the linear equation through the points (0, 70) (70 miles north of the transmitter) and (74, 0) (74 miles east):

$y = -\frac{35}{37}(x - 74) = -\frac{35}{37}x + 70$

The fraction is going to be cumbersome, but if we're going to approximate it on the calculator, we should use a number of decimal places:

$y = -0.945946x + 70$

Substituting y into the equation for the circle:

$x^2 + (-0.945946\, x + 70)^2 = 2809$

$x^2 + 0.894814x^2 - 132.43244x + 4900 = 2809$

$1.894814x^2 - 132.43244x + 2091 = 0$

Applying the quadratic formula, $x \cong 24.0977$ and $x \cong 45.7944$. The points of intersection (using the linear equation to get the y-values) are (24.0977, 47.2044) and (45.7944, 26.6810).

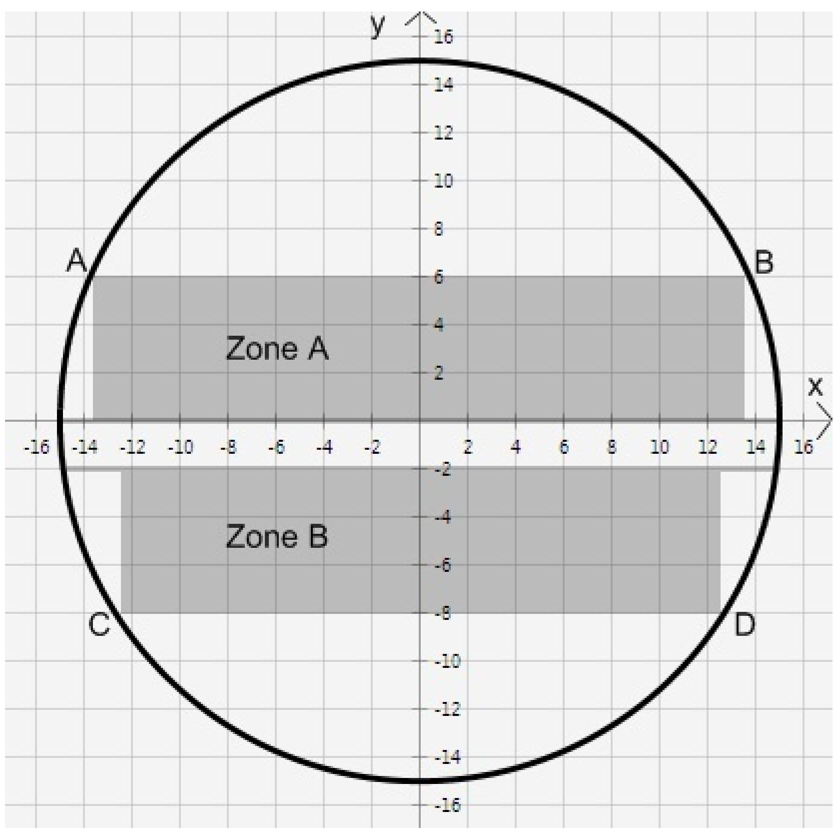
The distance between these two points is:

$d = \sqrt{(45.7944 - 24.0977)^2 + (26.6810 - 47.2044)^2} \approx 29.87$ miles.

19. Place the circular cross section in the Cartesian plane with center at (0, 0); the radius of the circle is 15 feet. This gives us the equation for the circle:

$x^2 + y^2 = 15^2$

$x^2 + y^2 = 225$



If we can determine the coordinates of points A, B, C and D, then the width of deck A's "safe zone" is the horizontal distance from point A to point B, and the width of deck B's "safe zone" is the horizontal distance from point C to point D.

The line connecting points A and B has the equation:

$y = 6$

Substituting $y = 6$ in the equation of the circle allows us to determine the x-coordinates of points A and B:

$x^2 + 6^2 = 225$

$x^2 = 189$

$x = \pm\sqrt{189}$, and $x \approx \pm 13.75$. Notice that this seems to agree with our drawing. Zone A stretches from $x \approx -13.75$ to $x \approx 13.75$, so its width is about 27.5 feet.

To determine the width of zone B, we intersect the line $y = -8$ with the equation of the circle:

$x^2 + (-8)^2 = 225$

$x^2 + 64 = 225$

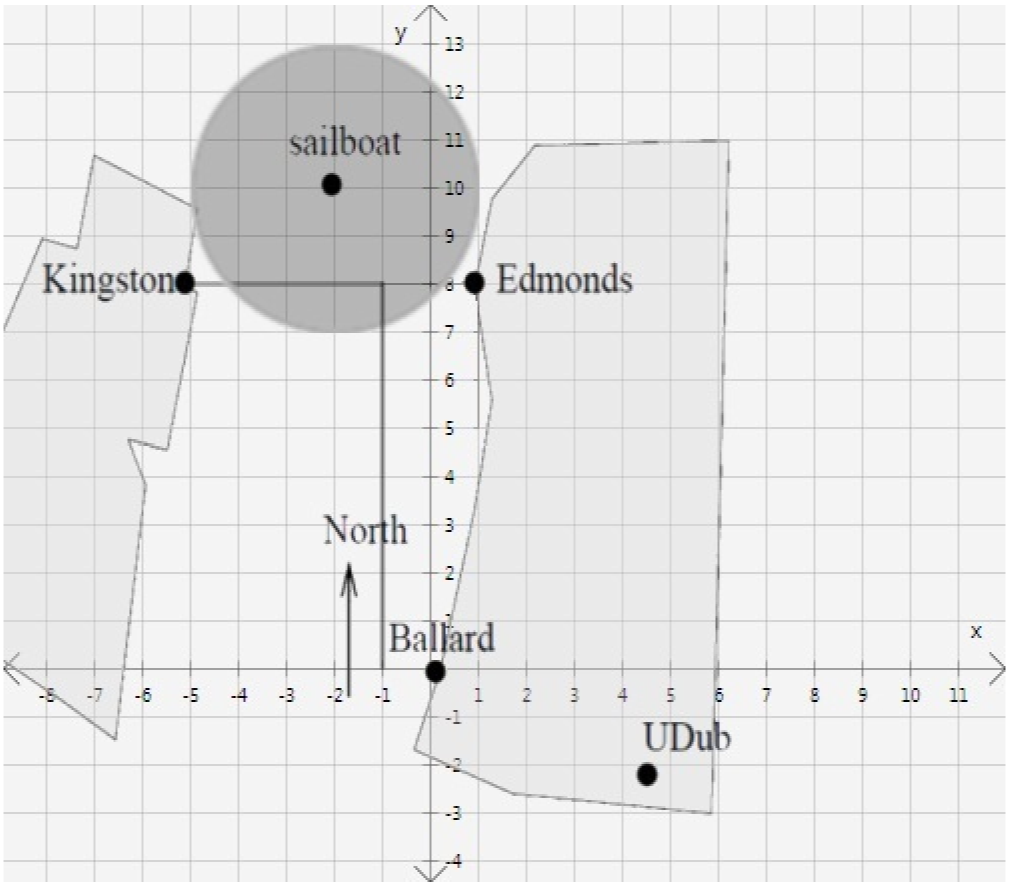
$x^2 = 161$

$x = \pm\sqrt{161}$

$x \approx \pm 12.69$

The width of zone B is therefore approximately 25.38 feet. Notice that this is less than the width of zone A, as we expect.

21.



Since Ballard is at the origin (0, 0), Edmonds must be at (1, 8) and Kingston at (-5, 8). Therefore, Eric's sailboat is at (-2, 10).

(a) Heading east from Kingston to Edmonds, the ferry's movement corresponds to the line $y = 8$. Since it travels for 20 minutes at 12 mph, it travels 4 miles, turning south at (-1, 8). The equation for the second line is $x = -1$.

(b) The boundary of the sailboat's radar zone can be described as $(x + 2)^2 + (y - 10)^2 = 3^2$; the interior of this zone is $(x + 2)^2 + (y - 10)^2 < 3^2$ and the exterior of this zone is $(x + 2)^2 + (y - 10)^2 > 3^2$.

(c) To find when the ferry enters the radar zone, we are looking for the intersection of the line $y = 8$ and the boundary of the sailboat's radar zone. Substituting $y = 8$ into the equation of the circle, we have $(x+2)^2 + (-2)^2 = 9$, and $(x+2)^2 = 5$. Therefore, $x + 2 = \pm\sqrt{5}$ and $x = -2 \pm \sqrt{5}$. These two values are approximately 0.24 and -4.24. The ferry enters at $x = -4.24$ ($x = 0.24$ is where it would have exited the radar zone, had it continued on toward Edmonds). Since it started at Kingston, which has an x-coordinate of -5, it has traveled about 0.76 miles. This journey – at 12 mph – requires about 0.0633 hours, or about 3.8 minutes.

(d) The ferry exits the radar zone at the intersection of the line $x = -1$ with the circle. Substituting, we have $1^2 + (y-10)^2 = 9$, $(y-10)^2 = 8$, and $y - 10 = \pm\sqrt{8}$. $y = 10 + \sqrt{8} \approx 12.83$ is the northern boundary of the intersection; we are instead interested in the southern boundary, which is at $y = 10 - \sqrt{8} \approx 7.17$. The ferry exits the radar zone at (-1, 7.17). It has traveled 4 miles from Kingston to the point at which it turned, plus an additional 0.83 miles heading south, for a total of 4.83 miles. At 12 mph, this took about 0.4025 hours, or 24.2 minutes.

(e) The ferry was inside the radar zone for all 24.2 minutes except the first 3.8 minutes (see part (c)). Thus, it was inside the radar zone for 20.4 minutes.

23. (a) The ditch is 20 feet high, and the water rises one foot (12 inches) in 6 minutes, so it will take 120 minutes (or two hours) to fill the ditch.

(b) Place the origin of a Cartesian coordinate plane at the bottom-center of the ditch. The four circles, from left to right, then have centers at (-40, 10), (-20, 10), (20, 10) and (40, 10) respectively:

$$(x+40)^2 + (y-10)^2 = 100$$

$$(x+20)^2 + (y-10)^2 = 100$$

$$(x-20)^2 + (y-10)^2 = 100$$

$$(x-40)^2 + (y-10)^2 = 100$$

Solving the first equation for y, we get:

$$(y-10)^2 = 100 - (x+40)^2$$

$$y - 10 = \pm\sqrt{100-(x+40)^2}$$

$$y = 10 \pm \sqrt{100-(x+40)^2}$$

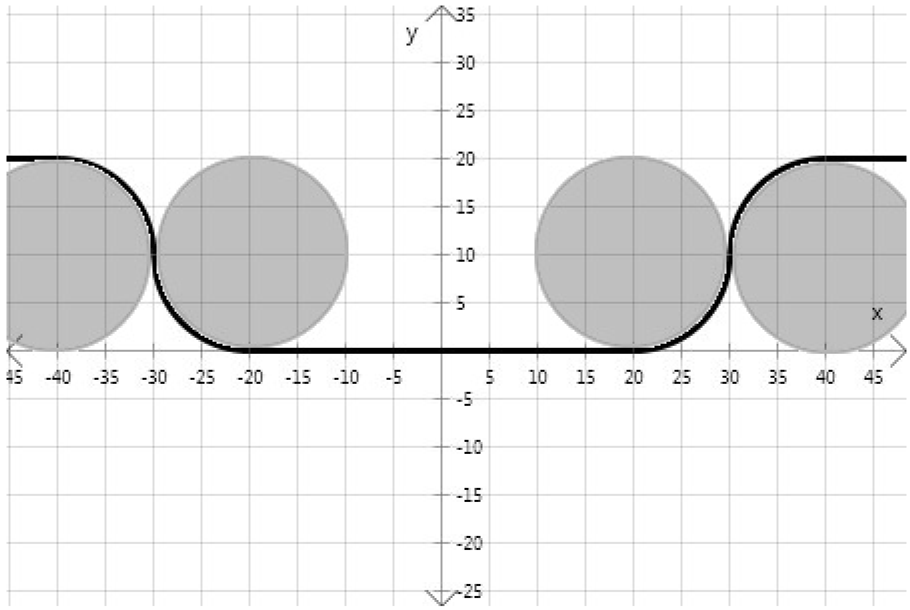
Since we are only concerned with the upper-half of this circle (actually, only the upper-right fourth of it), we can choose:

$$y = 10 + \sqrt{100-(x+40)^2}$$

A similar analysis will give us the desired parts of the other three circles. Notice that for the second and third circles, we need to choose the – part of the $\pm$ to describe (part of) the bottom half of the circle.

The remaining parts of the piecewise function are constant equations:

$$y = \begin{cases} 20 & x < -40 \\ 10 + \sqrt{100-(x+40)^2} & -40 \le x < -30 \\ 10 - \sqrt{100-(x+20)^2} & -30 \le x < -20 \\ 0 & -20 \le x < 20 \\ 10 - \sqrt{100-(x-20)^2} & 20 \le x < 30 \\ 10 + \sqrt{100-(x-40)^2} & 30 \le x < 40 \\ 20 & x \ge 40 \end{cases}$$



(c) At 1 hour and 18 minutes (78 minutes), the ditch will have 156 inches of water, or 13 feet. We need to find the x-coordinates that give us $y = 13$. Looking at the graph, it is clear that this occurs in the first and fourth circles. For the first circle, we return to the original equation:

$$(x + 40)^2 + (13 - 10)^2 = 100$$

$$(x + 40)^2 = 91$$

$$x = -40 \pm \sqrt{91}$$

We choose $x = -40 + \sqrt{91} \approx -30.46$ since x must be in the domain of the second piece of the piecewise function, which represents the upper-right part of the first circle. For the fourth (rightmost) circle, we have:

$$(x - 40)^2 + (13 - 10)^2 = 100$$

$$(x - 40)^2 = 91$$

$$x = 40 \pm \sqrt{91}$$

We choose $x = 40 - \sqrt{91} \approx 30.46$ to ensure that x is in the domain of the sixth piece of the piecewise function, which represents the upper-left part of the fourth circle.

The width of the ditch is therefore $30.46 - (-30.46) = 60.92$ feet. Notice the symmetry in the x-coordinates we found.

(d) Since our piecewise function is symmetrical across the y-axis, when the filled portion of the ditch is 42 feet wide, we can calculate the water height y using either $x = -21$ or $x = 21$. Using $x = 21$, we must choose the fifth piece of the piecewise function:

$$y = 10 - \sqrt{100 - (21 - 20)^2} = 10 - \sqrt{99} \approx 0.05$$

The height is 0.05 feet. At 6 minutes per foot, this will happen after 0.3 minutes, or 18 seconds.

(Notice that we could have chosen $x = -21$; then we would have used the third piece of the function and calculated:

$$10 - \sqrt{100 - (-21 + 20)^2} = 10 - \sqrt{99} \approx 0.05.\ )$$

When the width of the filled portion is 50 feet, we choose either $x = -25$ or $x = 25$. Choosing $x = 25$ requires us to use the fifth piece of the piecewise function:

$$10 - \sqrt{100 - (25 - 20)^2} = 10 - \sqrt{75} \approx 1.34$$

The height of the water is then 1.34 feet. At 6 minutes per foot, this will happen after 8.04 minutes.

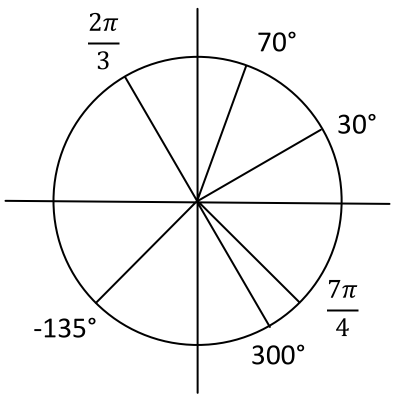
Finally, when the width of the filled portion is 73 feet, we choose $x = 36.5$. This requires us to use the sixth piece of the piecewise function:

$$10 + \sqrt{100 - (36.5 - 40)^2} = 10 + \sqrt{87.75} \approx 19.37$$

The height of the water is 19.37 feet after about 116.2 minutes have elapsed. At 19.37 feet, the ditch is nearly full; the answer to part a) told us that it is completely full after 120 minutes.

5.2 Solutions to Exercises

1.



3. $(180°)\left(\frac{\pi}{180°}\right) = \pi$

5. $\left(\frac{5\pi}{6}\right)\left(\frac{180°}{\pi}\right) = 150°$

7. $685° - 360° = 325°$

9. $-1746° + 5(360°) = 54°$

11. $\left(\frac{26\pi}{9} - 2\pi\right) = \left(\frac{26\pi}{9} - \frac{18\pi}{9}\right) = \frac{8\pi}{9}$

13. $\left(\frac{-3\pi}{2} + 2\pi\right) = \left(\frac{-3\pi}{2} + \frac{4\pi}{2}\right) = \frac{\pi}{2}$

15. $r = 7\text{ m}, \theta = 5\text{ rad}, s = \theta r \;\rightarrow\; s = (7\text{ m})(5) = 35\text{ m}$

17. $r = 12\text{ cm}, \theta = 120° = \frac{2\pi}{3}, s = \theta r \;\rightarrow\; s = (12\text{ cm})(\frac{2\pi}{3}) = 8\pi\text{cm}$

19. $r = 3960$ miles, $\theta = 5$ minutes $= \frac{5}{60}° = \frac{\pi}{2160}$, $s = r\theta \rightarrow s = (3960 \text{ miles})\left(\frac{\pi}{2160}\right) = \frac{11\pi}{6}$ miles

21. $r = 6$ ft, $s = 3$ ft, $s = r\theta \ \rightarrow \ \theta = s/r$

Plugging in we have $\theta = (3 \text{ ft})/(6 \text{ ft}) = 1/2$ rad.

$(1/2)((180°)/(\pi)) = 90/\pi° \approx 28.6479°$

23. $\theta = 45° = \frac{\pi}{4}$ radians, $r = 6$ cm

$A = \frac{1}{2}\theta r^2$; we have $A = \frac{1}{2}\left(\frac{\pi}{4}\right)(6 \text{ cm})^2 = \frac{9\pi}{2}$ cm^2

25. $D = 32$ in, speed = 60 mi/hr = 1 mi/min = 63360 in/min $= v$, $C = \pi D$, $v = \frac{s}{t}$, $s = \theta r$, $v = \frac{\theta r}{t}$, $\frac{v}{t} = \frac{\theta}{t} = \omega$. So $\omega = \frac{63360\frac{\text{in}}{\text{min}}}{16 \text{ in}}$ = 3960 rad/min. Dividing by 2π will yield 630.25 rotations per minute.

27. $r = 8$ in., $\omega = 15°/\text{sec} = \frac{\pi}{12}$ rad/sec, $v = \omega r$, $v = \left(\frac{\pi}{12}\right)(8 \text{ in}) = \frac{2.094 \text{ in}}{\text{sec}}$. To find RPM we must multiply by a factor of 60 to convert seconds to minutes, and then divide by 2π to get, RPM=2.5.

29. $d = 120$ mm for the outer edge. $\omega = 200$ rpm; multiplying by 2π rad/rev, we get $\omega = 400\pi$ rad/min. $v = r\omega$, and $r = 60$ mm, so $v = (60 \text{ mm})\left(400\pi \frac{\text{rad}}{\text{min}}\right) = 75{,}398.22 \text{mm/min}$. Dividing by a factor of 60 to convert minutes into seconds and then a factor of 1,000 to convert and mm into meters, this gives $v = 1.257$ m/sec.

31. $r = 3960$ miles. One full rotation takes 24 hours, so $\omega = \frac{\theta}{t} = \frac{2\pi}{24 \text{ hours}} = \frac{\pi}{12}$ rad/hour. To find the linear speed, $v = r\omega$, so $v = (3960 \text{ miles})\left(\frac{\pi}{12}\frac{\text{rad}}{\text{hour}}\right) = 1036.73 \text{miles/hour}$.

5.3 Solutions to Exercises

1. a. Recall that the sine is negative in quadrants 3 and 4, while the cosine is negative in quadrants 2 and 3; they are both negative only in quadrant III

   b. Similarly, the sine is positive in quadrants 1 and 2, and the cosine is negative in quadrants 2 and 3, so only quadrant II satisfies both conditions.

3. Because sine is the x-coordinate divided by the radius, we have $\sin\theta = \left(\frac{3}{5}\right)/1$ or just $\frac{3}{5}$. If we use the trig version of the Pythagorean theorem, $\sin^2\theta + \cos^2\theta = 1$, with $\sin\theta = \frac{3}{5}$, we get $\frac{9}{25} + \cos^2\theta = 1$, so $\cos^2\theta = \frac{25}{25} - \frac{9}{25}$ or $\cos^2\theta = \frac{16}{25}$; then $\cos\theta = \pm\frac{4}{5}$. Since we are in quadrant 2, we know that $\cos\theta$ is negative, so the result is $-\frac{4}{5}$.

5. If $\cos\theta = \frac{1}{7}$, then from $\sin^2\theta + \cos^2\theta = 1$ we have $\sin^2\theta + \frac{1}{49} = 1$, or $\sin^2\theta = \frac{49}{49} - \frac{1}{49} = \frac{48}{49}$. Then $\sin\theta = \pm\frac{\sqrt{48}}{7}$; simplifying gives $\pm\frac{4\sqrt{3}}{7}$ and we know that in the 4th quadrant $\sin\theta$ is negative, so our final answer is $-\frac{4\sqrt{3}}{7}$.

7. If $\sin\theta = \frac{3}{8}$ and $\sin^2\theta + \cos^2\theta = 1$, then $\frac{9}{64} + \cos^2\theta = 1$, so $\cos^2\theta = \frac{55}{64}$ and $\cos\theta = \frac{\pm\sqrt{55}}{8}$; in the second quadrant we know that the cosine is negative so the answer is $-\frac{\sqrt{55}}{8}$.

9. a. 225 is 45 more than 180, so our reference angle is 45°. 225° lies in quadrant III, where sine is negative and cosine is negative, then $\sin(225°) = -\sin(45°) = -\frac{\sqrt{2}}{2}$, and $\cos(225°) = -\cos(45°) = -\frac{\sqrt{2}}{2}$.

   b. 300 is 60 less than 360 (which is equivalent to zero degrees), so our reference angle is 60°. 300 lies in quadrant IV, where sine is negative and cosine is positive. $\sin(300°) = -\sin(60°) = -\frac{\sqrt{3}}{2}$; $\cos(300°) = \cos(60°) = \frac{1}{2}$.

   c. 135 is 45 less than 180, so our reference angle is 45°. 135° lies in quadrant II, where sine is positive and cosine is negative. $\sin(135°) = \sin(45°) = \frac{\sqrt{2}}{2}$; $\cos(135°) = -\cos(45°) =$

$-\frac{\sqrt{2}}{2}$.

d. 210 is 30 more than 180, so our reference angle is 30°. 210° lies in quadrant III, where sine and cosine are both negative. $\sin(210°) = -\sin(30°) = -\frac{1}{2}$; $\cos(210°) = -\cos(30°) = -\frac{\sqrt{3}}{2}$.

11. a. $\frac{5\pi}{4}$ is $\pi + \frac{\pi}{4}$ so our reference is $\frac{\pi}{4}$. $\frac{5\pi}{4}$ lies in quadrant III, where sine and cosine are both negative. $\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$; $\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

b. $\frac{7\pi}{6}$ is $\pi + \frac{\pi}{6}$ so our reference is $\frac{\pi}{6}$. $\frac{7\pi}{6}$ is in quadrant III where sine and cosine are both negative. $\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$; $\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$.

c. $\frac{5\pi}{3}$ is $2\pi - \frac{\pi}{3}$ so the reference angle is $\frac{\pi}{3}$, in quadrant IV where sine is negative and cosine is positive. $\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$; $\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

d. $\frac{3\pi}{4}$ is $\pi - \frac{\pi}{4}$; our reference is $\frac{\pi}{4}$, in quadrant II where sine is positive and cosine is negative. $\sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$; $\cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

13. a. $\frac{-3\pi}{4}$ lies in quadrant 3, and its reference angle is $\frac{\pi}{4}$, so $\sin\left(-\frac{3\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$; $\cos\left(-\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

b. $\frac{23\pi}{6} = \frac{12\pi}{6} + \frac{11\pi}{6} = 2\pi + \frac{11\pi}{6}$; we can drop the 2π, and notice that $\frac{11\pi}{6} = 2\pi - \frac{\pi}{6}$ so our reference angle is $\frac{\pi}{6}$, and $\frac{11\pi}{6}$ is in quadrant 4 where sine is negative and cosine is positive. Then $\sin\left(\frac{23\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$ and $\cos\left(\frac{23\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

c. For $\frac{-\pi}{2}$, if we draw a picture we see that the ray points straight down, so *y* is -1 and *x* is 0; $\sin\left(-\frac{\pi}{2}\right) = \frac{y}{1} = -1$; $\cos\left(-\frac{\pi}{2}\right) = \frac{x}{1} = 0$.

d. $5\pi = 4\pi + \pi = (2 * 2\pi) + \pi$; remember that we can drop multiples of 2π so this is the same as just π. $\sin(5\pi) = \sin(\pi) = 0$; $\cos(5\pi) = \cos(\pi) = -1$.

15. a. $\frac{\pi}{3}$ is in quadrant 1, where sine is positive; if we choose an angle with the same reference angle as $\frac{\pi}{3}$ but in quadrant 2, where sine is also positive, then it will have the same sine value. $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, so $\frac{2\pi}{3}$ has the same reference angle and sine as $\frac{\pi}{3}$.

b. Similarly to problem a. above, 100° = 180° - 80° , so both 80° and 100° have the same reference angle (80°) , and both are in quadrants where the sine is positive, so 100° has the same sine as 80°.

c. 140° is 40° less than 180°, so its reference angle is 40°. It is in quadrant 2, where the sine is positive; the sine is also positive in quadrant 1, so 40° has the same sine value and sign as 140°.

d. $\frac{4\pi}{3}$ is $\frac{\pi}{3}$ more than π, so its reference angle is $\frac{\pi}{3}$. It is in quadrant 3, where the sine is negative. Looking for an angle with the same reference angle of $\frac{\pi}{3}$ in a different quadrant where the sine is also negative, we can choose quadrant 4 and $\frac{5\pi}{3}$ which is $2\pi - \frac{\pi}{3}$.

e. 305° is 55° less than 360°, so its reference angle is 55° . It is in quadrant 4, where the sine is negative. An angle with the same reference angle of 55° in quadrant 3 where the sine is also negative would be 180° + 55° = 235°.

17. a. $\frac{\pi}{3}$ has reference angle $\frac{\pi}{3}$ and is in quadrant 1, where the cosine is positive. The cosine is also positive in quadrant 4, so we can choose $2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$.

b. 80° is in quadrant 1, where the cosine is positive, and has reference angle 80° . We can choose quadrant 4, where the cosine is also positive, and where a reference angle of 80° gives 360° - 80° = 280°.

c. 140° is in quadrant 2, where the cosine is negative, and has reference angle 180° - 140° = 40°. We know that the cosine is also negative in quadrant 3, where a reference angle of 40° gives 180° + 40° = 220°.

d. $\frac{4\pi}{3}$ is $\pi + \frac{\pi}{3}$, so it is in quadrant 3 with reference angle $\frac{\pi}{3}$. In this quadrant the cosine is negative; we know that the cosine is also negative in quadrant 2, where a reference angle of $\frac{\pi}{3}$ gives $\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$.

e. 305° = 360° - 55°, so it is in quadrant 4 with reference angle 55°. In this quadrant the cosine is positive, as it also is in quadrant 1, so we can just choose 55° as our result.

19. Using a calculator, $\cos(220°) \approx -0.76604$ and $\sin(220°) \approx -0.64279$. Plugging in these values for cosine and sine along with $r = 15$ into the formulas $\cos(\theta) = \frac{x}{r}$ and $\sin(\theta) = \frac{y}{r}$, we get the equations $-0.76604 \approx \frac{x}{15}$ and $-0.64279 \approx \frac{y}{15}$. Solving gives us the point $(-11.49067, -9.64181)$.

21. a. First let's find the radius of the circular track. If Marla takes 46 seconds at 3 meters/second to go around the circumference of the track, then the circumference is 46 · 3 = 138 meters. From the formula for the circumference of a circle, we have 138 = 2π*r*, so *r* = 138/2π ≈ 21.963 meters.

Next, let's find the angle between north (up) and her starting point; if she runs for 12 seconds, she covers 12/46 of the complete circle, which is 12/46 of 360° or about 0.261 · 360° ≈ 93.913° . The northernmost point is at 90° (since we measure angles from the positive *x*-axis) so her starting point is at an angle of 90° + 93.913° = 183.913°. This is in quadrant 3; we can get her *x* and *y* coordinates using the reference angle of 3.913°: $x = -r\cos(3.913°)$ and $y = -r\sin(3.913°)$. We get $(-21.9118003151, -1.4987914972)$.

b. Now let's find how many degrees she covers in one second of running; this is just 1/46 of 360° or 7.826°/sec. So, in 10 seconds she covers 78.26° from her starting angle of 183.913°. She's running clockwise, but we measure the angle counterclockwise, so we subtract to find that after 10 seconds she is at (183.913° - 78.26°) = 105.653°. In quadrant 2, the reference angle is 180° - 105.653° = 74.347°. As before, we can find her coordinates from $x = -r\cos(74.347°)$ and $y = r\sin(74.347°)$. We get $(-5.92585140281, 21.1484669457)$.

c. When Marla has been running for 901.3 seconds, she has gone around the track several times; each circuit of 46 seconds brings her back to the same starting point, so we divide 901.3 by 46 to get 19.5935 circuits, of which we only care about the last 0.5935 circuit; 0.5935 · 360° = 213.652° measured clockwise from her starting point of 183.913°, or 183.913° - 213.652° = -29.739°. We can take this as a reference angle in quadrant 4, so her coordinates are $x = r\cos(29.739°)$ and $y = -r\sin(29.739°)$. We get $(19.0703425544, -10.8947420281)$.

Last edited 1/29/13

5.4 Solutions to Exercises

1. $\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$; $\csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$; $\tan\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$; $\cot\left(\frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{4}\right)} = 1$

3. $\sec\left(\frac{5\pi}{6}\right) = \frac{1}{\cos\left(\frac{5\pi}{6}\right)} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$; $\csc\left(\frac{5\pi}{6}\right) = \frac{1}{\sin\left(\frac{5\pi}{6}\right)} = \frac{1}{\frac{1}{2}} = 2$; $\tan\left(\frac{5\pi}{6}\right) = \frac{\sin\left(\frac{5\pi}{6}\right)}{\cos\left(\frac{5\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{-(\sqrt{3})}{2}} = -\frac{\sqrt{3}}{3}$; $\cot\left(\frac{5\pi}{6}\right) = \frac{1}{\tan\left(\frac{5\pi}{6}\right)} = \frac{1}{-\frac{\sqrt{3}}{3}} = -\sqrt{3}$

5. $\sec\left(\frac{2\pi}{3}\right) = \frac{1}{\cos\left(\frac{2\pi}{3}\right)} = \frac{1}{\frac{-1}{2}} = -2$; $\csc\left(\frac{2\pi}{3}\right) = \frac{1}{\sin\left(\frac{2\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$; $\tan\left(\frac{2\pi}{3}\right) = \frac{\sin\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = -\sqrt{3}$; $\cot\left(\frac{2\pi}{3}\right) = \frac{1}{\tan\left(\frac{2\pi}{3}\right)} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

7. $\sec(135°) = \frac{1}{\cos(135°)} = \frac{1}{\frac{-\sqrt{2}}{2}} = -\sqrt{2}$; $\csc(210°) = \frac{1}{\sin(210°)} = \frac{1}{\frac{-1}{2}} = -2$; $\tan(60°) = \frac{\sin(60°)}{\cos(60°)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$; $\cot(225°) = \frac{1}{\tan(225°)} = \frac{1}{1} = 1$

9. Because θ is in quadrant II, we know $\cos(\theta) < 0, \sec(\theta) = \frac{1}{\cos(\theta)} < 0;\ \sin(\theta) > 0, \csc(\theta) = \frac{1}{\sin(\theta)} > 0; \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} < 0, \cot(\theta) = \frac{1}{\tan(\theta)} < 0.$

Then: $\cos(\theta) = -\sqrt{1-\sin^2(\theta)} = -\sqrt{1-\left(\frac{3}{4}\right)^2} = -\frac{\sqrt{7}}{4}$; $\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\frac{-\sqrt{7}}{4}} = -\frac{4\sqrt{7}}{4}$;

$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$; $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{3}{4}}{\frac{-\sqrt{7}}{4}} = \frac{-3\sqrt{7}}{7}$; $\cot(\theta) = \frac{1}{\tan\theta} = \frac{1}{\frac{-3\sqrt{7}}{7}} = \frac{7}{-3\sqrt{7}} = -\frac{\sqrt{7}}{3}$

11. In quadrant III, $x < 0,\ y < 0$. Imaging a circle with radius r,

$\sin(\theta) = \frac{y}{r} < 0, \csc(\theta) = \frac{1}{\sin(\theta)} < 0; \cos(\theta) = \frac{x}{r} < 0, sec(\theta) = \frac{1}{\cos(\theta)} < 0; \tan(\theta) = \frac{x}{y} > 0, \cot(\theta) = \frac{1}{\tan(\theta)} > 0.$

Then: $\sin(\theta) = -\sqrt{1-\cos^2(\theta)} = \frac{-2\sqrt{2}}{3}; \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{3}{-2\sqrt{2}} = -\frac{3\sqrt{2}}{4}; \sec(\theta) = \frac{1}{\cos(\theta)} = -3; \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = 2\sqrt{2};\ \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

13. $0 \le \theta \le \frac{\pi}{2}$ means θ is in the first quadrant, so $x > 0, y >$. In a circle with radius r:

$\sin(\theta) = \frac{y}{r} > 0, \csc(\theta) = \frac{1}{\sin(\theta)} > 0; \cos(\theta) = \frac{x}{r} > 0, \sec(\theta) = \frac{1}{\cos(\theta)} > 0; \cot(\theta) = \frac{1}{\tan(\theta)} > 0.$

Since $\tan(\theta) = \frac{y}{x} = \frac{12}{5}$, we can use the point (5, 12), for which $r = \sqrt{12^2 + 5^2} = 13$.

Then: $\sin(\theta) = \frac{y}{r} = \frac{12}{13}; \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{13}{12}; \cos(\theta) = \frac{x}{r} = \frac{5}{13}; \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{13}{5}; \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{5}{12}$

15. a. $\sin(0.15) = 0.1494;\ \cos(0.15) = 0.9888; \tan(0.15) = 0.15$

b. $\sin(4) = -0.7568;\ \cos(4) = -0.6536;\ \tan(4) = 1.1578$

c. $\sin(70°) = 0.9397;\ \cos(70°) = 0.3420;\ \tan(70°) = 2.7475$

d. $\sin(283°) = -0.9744; \cos(283°) = 0.2250 \tan(283°) = -4.3315$

17. $\csc(t)\tan(t) = \frac{1}{\sin(t)} \cdot \frac{\sin(t)}{\cos(t)} = \frac{1}{\cos(t)} = \sec(t)$

19. $\frac{\sec(t)}{\csc(t)} = \frac{\frac{1}{\cos(t)}}{\frac{1}{\sin(t)}} = \frac{\sin(t)}{\cos(t)} = \tan(t)$

21. $\frac{\sec(t) - \cos(t)}{\sin(t)} = \frac{\frac{1}{\cos(t)} - \cos(t)}{\sin(t)} = \frac{1-\cos^2(t)}{\sin(t)\cos(t)} = \frac{\sin^2(t)}{\sin(t)\cos(t)} = \frac{\sin(t)}{\cos(t)} = \tan(t)$

23. $\frac{1+\cot(t)}{1+\tan(t)} = \frac{1+\frac{1}{\tan(t)}}{1+\tan(t)} \cdot \frac{\tan(t)}{\tan(t)} = \frac{\tan(t)+1}{(1+\tan(t))\tan(t)} = \frac{1}{\tan(t)} = \cot(t)$

25. $\frac{\sin^2(t)+\cos^2(t)}{\cos^2(t)} = \frac{1}{\cos^2(t)} = \sec^2(t)$

27. $\frac{\sin^2(\theta)}{1+\cos(\theta)} = \frac{1-\cos^2(\theta)}{1+\cos(\theta)}$ by the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$= \frac{(1+\cos(\theta))(1-\cos(\theta))}{1+\cos(\theta)}$ by factoring

$= 1 - \cos(\theta)$ by reducing

29. $\sec(a) - \cos(a) = \frac{1}{\cos(a)} - \cos(a)$

$= \frac{1}{\cos(a)} - \frac{\cos^2(a)}{\cos(a)}$

$= \frac{\sin^2(a)}{\cos(a)}$

$= \sin(a) \cdot \frac{\sin(a)}{\cos(a)}$

$= \sin(a) \cdot \tan(a)$

31. Note that (with this and similar problems) there is more than one possible solution. Here's one:

$\frac{\csc^2(x) - \sin^2(x)}{\csc(x)+ \sin(x)} = \frac{(\csc(x)+ \sin(x))(\csc(x) - \sin(x))}{\csc(x)+ \sin(x)}$ by factoring

$= \csc(x) - \sin(x)$ by reducing

$= \frac{1}{\sin(x)} - \sin(x)$

$= \frac{1}{\sin(x)} - \frac{\sin^2(x)}{\sin(x)}$

$= \frac{\cos^2(x)}{\sin(x)}$

$= \cos(x) \cdot \frac{\cos(x)}{\sin(x)}$

$= \cos(x) \cot(x)$

33. $\frac{\csc^2(\alpha)-1}{\csc^2(\alpha)-\csc(\alpha)} = \frac{(\csc(\alpha)+1)(\csc(\alpha)-1)}{\csc(\alpha)\cdot(\csc(\alpha)-1)}$

$= \frac{\csc(\alpha)+1}{\csc(\alpha)}$

$= \frac{\frac{1}{\sin(\alpha)}+1}{\frac{1}{\sin(\alpha)}}$ applying the identity $\csc(\alpha) = \frac{1}{\sin(\alpha)}$

$= \left(\frac{1}{\sin(\alpha)} + 1\right) \cdot \frac{\sin(\alpha)}{1}$

$= \frac{\sin(\alpha)}{\sin(\alpha)} + \frac{\sin(\alpha)}{1}$

$= 1 + \sin(\alpha)$

35. To get $\sin(u)$ into the numerator of the left side, we'll multiply the top and bottom by $1 - \cos(c)$ and use the Pythagorean identity:

$\frac{1+\cos(u)}{\sin(u)} \cdot \frac{1-\cos(u)}{1-\cos(u)} = \frac{1-\cos^2(u)}{\sin(u)(1-\cos(u))}$

$= \frac{\sin^2(u)}{\sin(u)(1-\cos(u))}$

$= \frac{\sin(u)}{1-\cos(u)}$

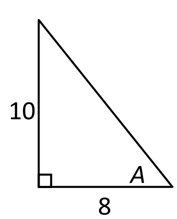
37. $\frac{\sin^4(\gamma)-\cos^4(\gamma)}{\sin(\gamma)-\cos(\gamma)} = \frac{(\sin^2(\gamma)+\cos^2(\gamma))\,(\sin^2(\gamma)-\cos^2(\gamma))}{\sin(\gamma)-\cos(\gamma)}$ by factoring

$= \frac{1\cdot(\sin(\gamma)+\cos(\gamma))(\sin(\gamma)-\cos(\gamma))}{\sin(\gamma)-\cos(\gamma)}$ by applying the Pythagorean identity, and factoring

$= \sin(\gamma) + \cos(\gamma)$ by reducing

5.5 Solutions to Exercises

1.



hypotenuse^2 = 10^2 + 8^2 = 164 => hypotenuse = $\sqrt{164}$ = $2\sqrt{41}$

Therefore, sin (A) = $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{10}{2\sqrt{41}} = \frac{5}{\sqrt{41}}$

cos (A) = $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{2\sqrt{41}} = \frac{4}{\sqrt{41}}$

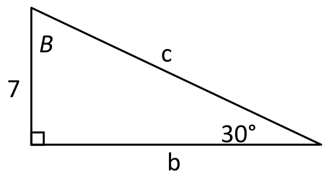
tan (A) = $\frac{\sin(A)}{\cos(A)} = \frac{\frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}}} = \frac{5}{4}$ or tan (A) = $\frac{\text{opposite}}{\text{adjacent}} = \frac{10}{8} = \frac{5}{4}$

$$\sec(A) = \frac{1}{\cos(A)} = \frac{1}{\frac{4}{\sqrt{41}}} = \frac{\sqrt{41}}{4}$$

$$\csc(A) = \frac{1}{\sin(A)} = \frac{1}{\frac{5}{\sqrt{41}}} = \frac{\sqrt{41}}{5}$$

and $\cot(A) = \frac{1}{\tan(A)} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$.

3.



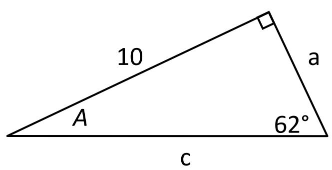
$\sin(30°) = \frac{7}{c} \Rightarrow c = \frac{7}{\sin(30°)} = \frac{7}{\frac{1}{2}} = 14$

$\tan(30°) = \frac{7}{b} \Rightarrow b = \frac{7}{\tan(30°)} = \frac{7}{\frac{1}{\sqrt{3}}} = 7\sqrt{3}$

or $7^2 + b^2 = c^2 = 14^2 \Rightarrow b^2 = 14^2 - 7^2 = 147 \Rightarrow b = \sqrt{147} = 7\sqrt{3}$

$\sin(B) = \frac{b}{c} = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2} \Rightarrow B = 60°$ or $B = 90° - 30° = 60°$.

5.

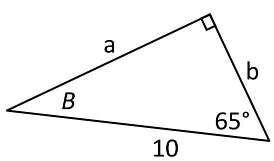


$\sin(62°) = \frac{10}{c} \Rightarrow c = \frac{10}{\sin(62°)} \approx 11.3257$

$\tan(62°) = \frac{10}{a} \Rightarrow a = \frac{10}{\tan(62°)} \approx 5.3171$

$A = 90° - 62° = 28°$

7.

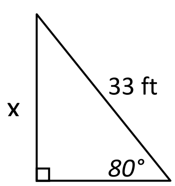


$B = 90° - 65° = 25°$

$\sin(B) = \sin(25°) = \frac{b}{10} \Rightarrow b = 10\sin(25°) \approx 4.2262$

$\cos(B) = \cos(25°) = \frac{a}{10} \Rightarrow a = 10\cos(25°) \approx 9.0631$

9.

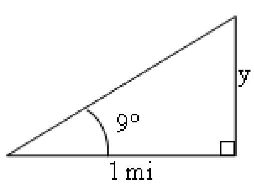


Let x (feet) be the height that the ladder reaches up.

Since $\sin(80°) = \frac{x}{33}$

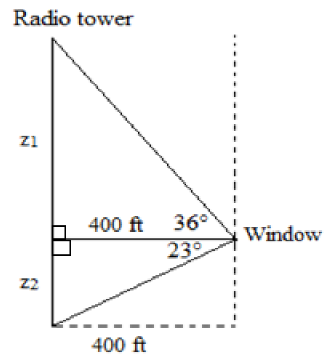
So the ladder reaches up to $x = 33\sin(80°) \approx 32.4987$ ft of the building.

11.



Let y (miles) be the height of the building. Since tan (9°) = $\frac{y}{1}$ = y, the height of the building is y = tan (9°) mi ≈ 836.26984 ft.

13.



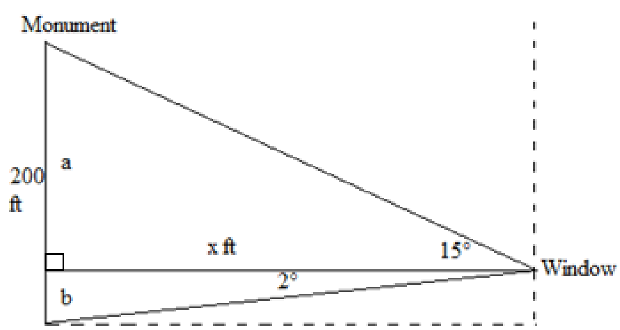
Let z_1 (feet) and z_2 (feet) be the heights of the upper and lower parts of the radio tower. We have

tan (36°) = $\frac{z_1}{400}$ => z_1 = 400 tan (36°) ft

tan (23°) = $\frac{z_2}{400}$ => z_2 = 400 tan (23°) ft

So the height of the tower is $z_1 + z_2$ = 400 tan (36°) + 400 tan (23°) ≈ 460.4069 ft.

15.



Let x (feet) be the distance from the person to the monument, *a* (feet) and *b* (feet) be the heights of the upper and lower parts of the building. We have

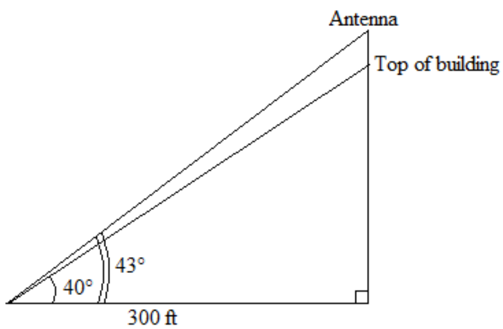
tan (15°) = $\frac{a}{x}$ => $a = x$ tan (15°)

and tan (2°) = $\frac{b}{x}$ => $b = x$ tan (2°)

Since $200 = a + b = x \tan(15°) + x \tan(2°) = x[\tan(15°) + \tan(2°)]$

Thus the distance from the person to the monument is $x = \frac{200}{\tan(15°) + \tan(2°)} \approx 660.3494$ ft.

17.

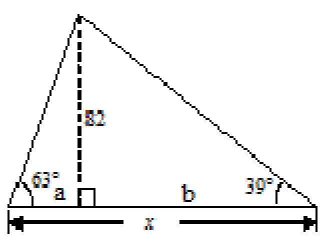


Since $\tan(40°) = \frac{\text{height from the base to the top of the building}}{300}$, the height from the base to the top of the building is $300 \tan(40°)$ ft.

Since $\tan(43°) = \frac{\text{height from the base to the top of the antenna}}{300}$, the height from the base to the top of the antenna is $300 \tan(43°)$ ft.

Therefore, the height of the antenna = $300 \tan(43°) - 300 \tan(40°) \approx 28.0246$ ft.

19.

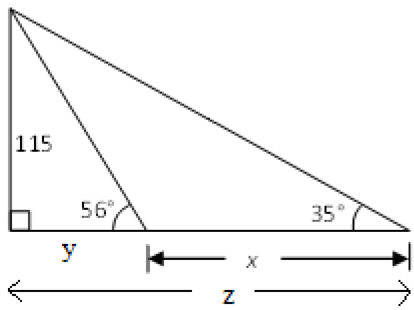


We have $\tan(63°) = \frac{82}{a} \Rightarrow a = \frac{82}{\tan(63°)}$

$\tan(39°) = \frac{82}{b} \Rightarrow b = \frac{82}{\tan(39°)}$

Therefore $x = a + b = \frac{82}{\tan(63°)} + \frac{82}{\tan(39°)} \approx 143.04265$.

21.

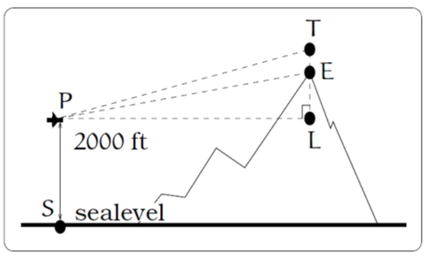


We have $\tan(35°) = \frac{115}{z} \Rightarrow z = \frac{115}{\tan(35°)}$

$\tan(56°) = \frac{115}{y} \Rightarrow y = \frac{115}{\tan(56°)}$

Therefore $x = z - y = \frac{115}{\tan(35°)} - \frac{115}{\tan(56°)} \approx 86.6685$.

23.

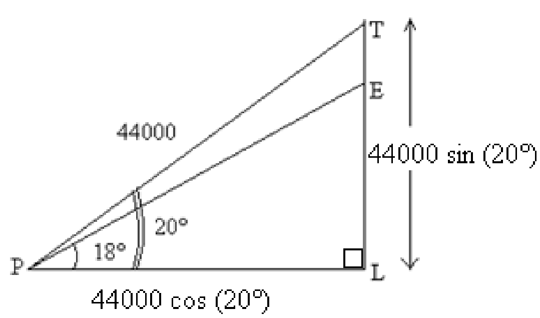


The length of the path that the plane flies from P to T is

$$PT = \left(\frac{100\ mi}{1\ h}\right)\left(\frac{1\ h}{60\ min}\right)(5\ min) = \frac{25}{3}\ mi = 44000\ ft$$

In ΔPTL, $\sin(20°) = \frac{TL}{PT} \Rightarrow TL = PT\sin(20°) = 44000\sin(20°)$ ft

$\cos(20°) = \frac{PL}{PT} \Rightarrow PL = PT\cos(20°) = 44000\cos(20°)$ ft



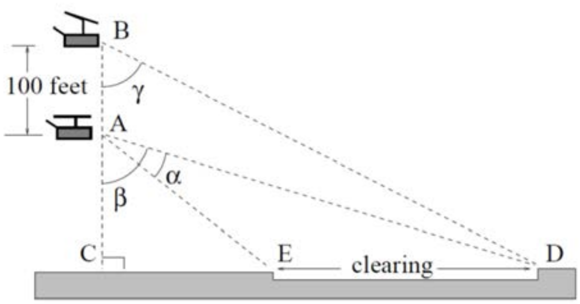
In ΔPEL, $\tan(18°) = \frac{EL}{PL} \Rightarrow EL = PL\tan(18°) = 44000\cos(20°)\tan(18°) \approx 13434.2842$ ft

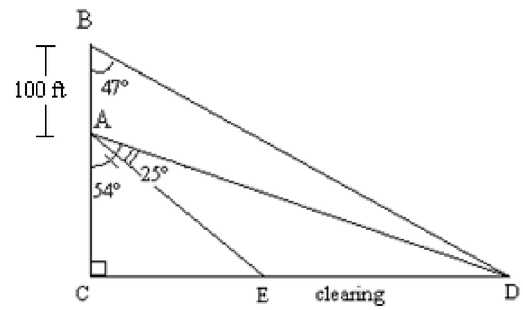
Therefore $TE = TL - EL = 44000\sin(20°) - 44000\cos(20°)\tan(18°)$

$= 44000[\sin(20°) - \cos(20°)\tan(18°)] \approx 1614.6021$ ft

So the plane is about 1614.6021 ft above the mountain's top when it passes over. The height of the mountain is the length of *EL*, about 13434.2842 ft, plus the distance from sea level to point *L*, 2000 ft (the original height of the plane), so the height is about 15434.2842 ft.

25.





We have: $\tan(47°) = \frac{CD}{BC} = \frac{CD}{AC+100}$ => $AC + 100 = \frac{CD}{\tan(47°)}$ => $AC = \frac{CD}{\tan(47°)} - 100$

$\tan(54°) = \frac{CD}{AC}$ => $AC = \frac{CD}{\tan(54°)}$

Therefore, $\frac{CD}{\tan(47°)} - 100 = \frac{CD}{\tan(54°)}$

$$\frac{CD}{\tan(47°)} - \frac{CD}{\tan(54°)} = 100$$

$$CD\left(\frac{1}{\tan(47°)} - \frac{1}{\tan(54°)}\right) = 100 \quad \text{or} \quad CD\left(\frac{\tan(54°)-\tan(47°)}{\tan(47°)\tan(54°)}\right) = 100$$

So $CD = \frac{100\tan(54°)\tan(47°)}{\tan(54°) - \tan(47°)}$

Moreover, $\tan(54° - 25°) = \tan(29°) = \frac{CE}{AC} = \frac{CE}{\frac{CD}{\tan(54°)}} = \frac{CE\tan(54°)}{CD}$

$$= \frac{CE\tan(54°)}{\frac{100\tan(54°)\tan(47°)}{\tan(54°)-\tan(47°)}} = \frac{CE[\tan(54°) - \tan(47°)]}{100\tan(47°)}$$

⇨ $CE = \frac{100\tan(47°)\tan(29°)}{\tan(54°)-\tan(47°)}$

Thus the width of the clearing should be $ED = CD - CE = \frac{100\tan(54°)\tan(47°)}{\tan(54°)-\tan(47°)} - \frac{100\tan(47°)\tan(29°)}{\tan(54°)-\tan(47°)}$

$$= \frac{100\tan(47°)}{\tan(54°)-\tan(47°)}[\tan(54°) - \tan(29°)] \approx 290 \text{ ft.}$$