Last edited 4/20/15

2.1 Solutions to Exercises

1. $P(t) = 1700t + 45{,}000$ 3. $D(t) = 2t + 10$

5. Timmy will have the amount $A(n)$ given by the linear equation $A(n) = 40 - 2n$.

7. From the equation, we see that the slope is 4, which is positive, so the function is increasing.

9. From the equation, we see that the slope is -2, which is negative, so the function is decreasing.

11. From the equation, we see that the slope is -2, which is negative, so the function is decreasing.

13. From the equation, we see that the slope is $\frac{1}{2}$, which is positive, so the function is increasing.

15. From the equation, we see that the slope is $-\frac{1}{3}$, which is negative, so the function is decreasing.

17. $m = \frac{10-4}{4-2} = \frac{6}{2} = 3$. 19. $m = \frac{2-4}{5-(-1)} = \frac{-2}{6} = -\frac{1}{3}$

21. $m = \frac{3-11}{-4-6} = \frac{-8}{-10} = \frac{4}{5}$ 23. $m = \frac{2}{3}$

25. $m = \frac{1.4-0.9}{12-2} = \frac{0.5\ miles}{10\ minutes} = \frac{.005\ miles}{1\ minute}$

27. $m = \frac{275{,}900-287{,}500}{1989-1960} = \frac{-11{,}600\ people}{29\ years} = \frac{-400\ people}{year}$. The negative rate means that the population is declining by approximately 400 people per year.

29. The rate is equal to the slope, which is 0.1. The initial value is the y-intercept, which is 24. This means that the phone company charges 0.1 dollars per minute, or 10 cents a minute, plus an additional fixed 24 dollars per month.

31. Terry starts skiing at 3000 feet, and skis downhill at a constant rate of 70 feet per second.

33. From this information we can extract two ordered pairs, $(-5, -4)$ and $(5,2)$. The slope between these two points is $m = \frac{2-(-4)}{5-(-5)} = \frac{6}{10} = \frac{3}{5}$. This gives us the formula $f(x) = \frac{3}{5}x + b$. To find the y-intercept b, we can substitute one of our ordered pairs into the equation for $f(x)$ and x. For example: $2 = \frac{3}{5}(5) + b$. Solving for b gives us $b = -1$. So, the final equation is $f(x) = \frac{3}{5}x - 1$.

35. The slope between these two points is $m = \frac{10-4}{4-2} = \frac{6}{2} = 3$. This gives us the formula $f(x) = 3x + b$. To find the y-intercept b, we can substitute one of our ordered pairs into the equation for $f(x)$ and x. For example: $4 = 3(2) + b$. Solving for b gives us $b = -2$. So, the final equation is $f(x) = 3x - 2$.

37. The slope between these two points is $m = \frac{2-4}{5-(-1)} = \frac{-2}{6} = -\frac{1}{3}$. This gives us the formula $f(x) = -\frac{1}{3}x + b$. To find the y-intercept b, we can substitute one of our ordered pairs into the equation for $f(x)$ and x. For example: $2 = -\frac{1}{3}(5) + b$. Solving for b gives us $b = \frac{11}{3}$. So, the final equation is $f(x) = -\frac{1}{3}x + \frac{11}{3}$.

39. The slope between these two points is $m = \frac{-3-0}{0-(-2)} = \frac{-3}{2} = -\frac{3}{2}$. We are given the y-intercept $b = -3$. So, the final equation is $f(x) = -\frac{3}{2}x - 3$.

41. $f(x) = \frac{2}{3}x + 1$ 43. $f(x) = -2x + 3$

45. From this information we can extract two ordered pairs, $(1000,30)$ and $(3000,22)$. The slope between these two points is $m = \frac{22-30}{3000-1000} = \frac{-8}{2000} = -\frac{1}{250}$. This gives us the formula $f(x) = -\frac{1}{250}x + b$. To find the y-intercept b, we can substitute one of our ordered pairs into the equation for $f(x)$ and x. For example: $30 = -\frac{1}{250}(1000) + b$. Solving for b gives us $b = 34$. So, the final equation is $f(x) = -\frac{1}{250}x + 34$.

47. (a) Linear, because x is changing at a constant rate, and $g(x)$ is also changing at a constant rate. The output is changing by -15, and the input is changing by 5. So, the rate of change is $-\frac{15}{5} = -3$. The y-intercept is given from the table as the ordered pair (0,5), so $b = 5$. So, the final equation is $g(x) = -3x + 5$.

    (b) Not linear, because $h(x)$ is not increasing a constant rate.

    (c) Linear, because x is changing at a constant rate, and $f(x)$ is also changing at a constant rate. The output is changing by 25, and the input is changing by 5. So, the rate of change is $\frac{25}{5} = 5$. The y-intercept is given from the table as the ordered pair (0,-5), so $b = -5$. So, the final equation is $f(x) = 5x - 5$.

    (d) Not linear, because $k(x)$ is not increasing a constant rate.

49. (a) From this information we can extract two points, (32,0) and (212,100) using F as the input and C as the output. The slope between these two points is $m = \frac{100-0}{212-32} = \frac{100}{180} = \frac{10}{18} = \frac{5}{9}$. This gives us the formula $C(F) = \frac{5}{9}F + b$. To find the y-intercept b, we can substitute one of our ordered pairs into the equation. For example: $0 = \frac{5}{9}(32) + b$. Solving for b gives us $b = -\frac{160}{9}$. So, the final equation is $C = \frac{5}{9}F - \frac{160}{9}$.

    (b) This can be done by solving the equation we found in part (a) for F instead of C. So, $F = \frac{9}{5}\left(C + \frac{160}{9}\right)$.
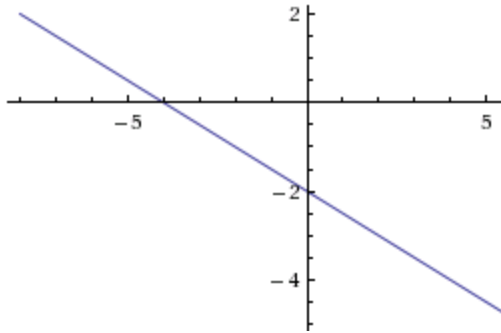
    (c) To find -23 C in Fahrenheit, we plug it into this equation for C and solve for F, giving us $F = -9.3$ degrees F
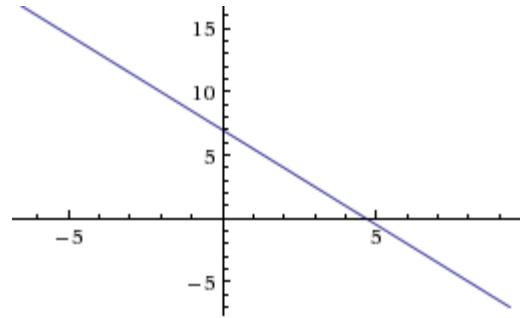
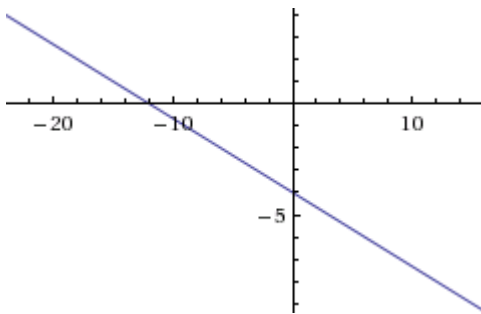## 2.2 Solutions to Exercises

1. E   3. D   5. B
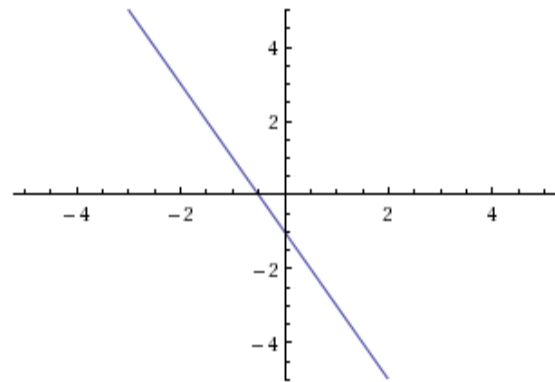
7. $f(x) = -\frac{1}{2}x - 2$

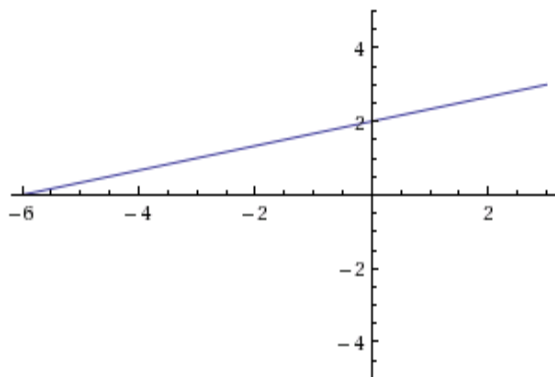9. $f(x) = -\frac{3}{2}x + 7$

11. $f(x) = -\frac{1}{3}x - 4$

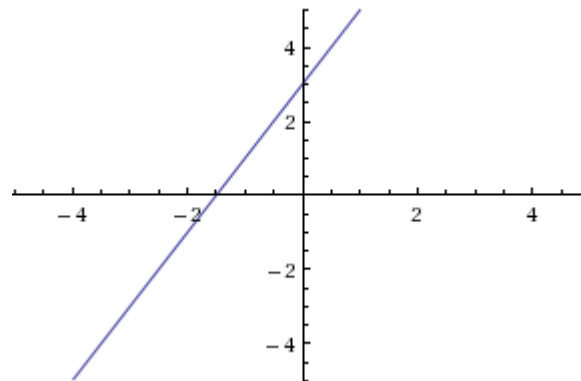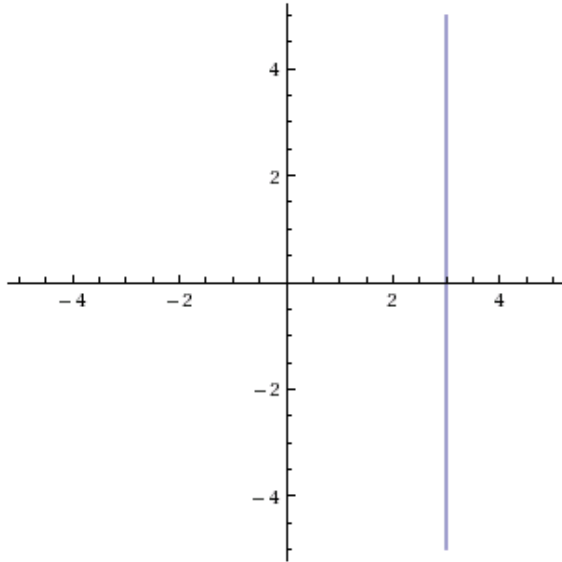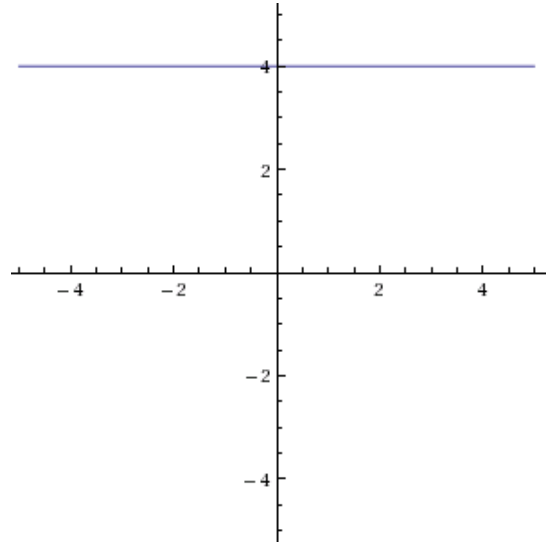13. $f(x) = -2x - 1$

15. $h(x) = \frac{1}{3}x + 2$

17. $k(t) = 3 + 2t$

19. $x = 3$ 21. $r(x) = 4$

23.   (a) $g(x) - 4 = \frac{3}{4}x + 2$ *or* $g(x) = \frac{3}{4}x + 6$

(b) $m = \frac{3}{4}$

(c) $b = 6$

25. $f(x) = 3$ 27. $x = -3$

29. Horizontal intercept (when $f(x) = 0$) $= -2$, vertical intercept (when $x = 0$) $= 2$.

31. Horizontal intercept (when $f(x) = 0$) $= \frac{5}{3}$, vertical intercept (when $x = 0$) $= -5$.

33. Horizontal intercept (when $f(x) = 0$) $= -10$, vertical intercept (when $x = 0$) $= 4$

35. Slope of line 1 $= -10$, Slope of line 2 $= -10$. They have the same slope, so they are parallel.

37. Slope of line 1 $= -\frac{1}{2}$, Slope of line 2 $= 1$. They are neither perpendicular or parallel.

39. Slope of line 1 $= -\frac{2}{3}$, Slope of line 2 $= \frac{3}{2}$. The product of the two slopes is -1, so they are perpendicular.

41. A line parallel to the graph of $f(x) = -5x - 3$ will have the same slope as $f(x)$; its slope is -5. Then the equation has the form $y = -5x + b$. Plugging in the given point $(2, -12)$, we get $-12 = (-5)(2) + b$. Solving this equation, we get $b = -2$, so the desired equation is $y = -5x - 2$.

43. A line perpendicular to $h(t)$ has a slope which is the opposite reciprocal of -2, which is $\frac{1}{2}$.

So it's equation has the form $f(x) = \frac{1}{2}x + b$. Plugging in the given point (-4,1) allows us

to solve for b, which equals 3. So, the final equation is $y = \frac{1}{2}x + 1$.

45. At the point where the two lines intersect, they will have the same y value, so we can set
$f(x)$ and $g(x)$ equal to each other. So, $-2x - 1 = -x$. Solving for x gives $x = -1$. To
find the y value, we plug this value for x back into either equation, which gives $y = 1$.
So, the point that the two lines intersect is
$(-1,1)$. *Note: this point can also be found by graphing.*

47. At the point where the two lines intersect, they will have the same y value, so we can set
$f(x)$ and $g(x)$ equal to each other. So, $-\frac{4}{5}x + \frac{274}{25} = \frac{9}{4}x + \frac{73}{10}$. Solving for x gives
$x = 1.2$. To find the
y value, we plug this value for x back into either equation, which gives $y = 10$. So, the
point that the two lines intersect is $(-1.2,10)$.

49. Plan A can be modeled by the linear equation
$A(m) = 30 + .18m$, and Plan B can be
represented by the fixed equation $B(m) = 50$
As the graph shows, the cost of renting a car i
cheaper with Plan A, until a certain point. This
is the point of intersection, which can be
found algebraically or graphically. This point
is (111.11, 50) meaning that at around 111 miles,
it is cheaper to go with Plan B.
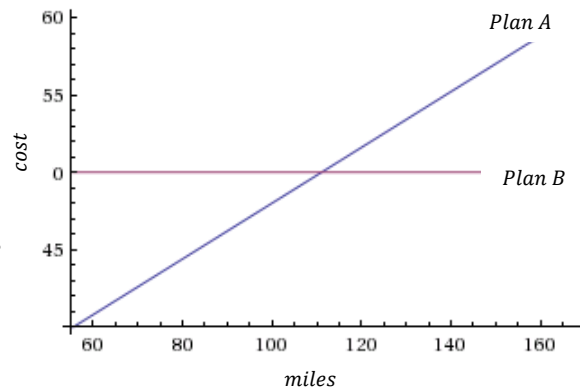
51. (a)
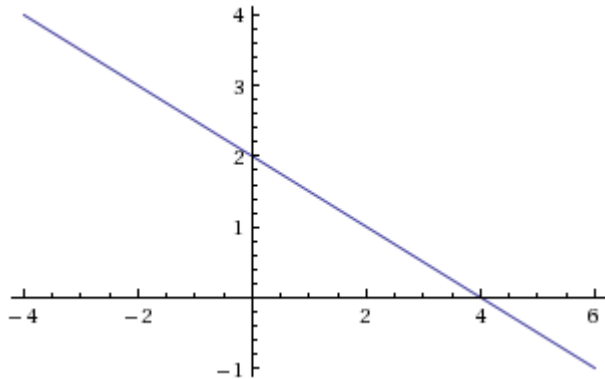
To find the point of intersection algebraically, we set the two lines equal to each other and solve for x. So, $2 - \frac{1}{2}x = 1 + cx$. This gives us $x = \frac{1}{c} + 2$. Plugging this back into either equation gives the y value $y = 2 + 2c$. So the point of intersection is $\left(\frac{1}{c} + 2, 2 + 2c\right)$

(b) The x-coordinate has the form $x = \frac{1}{c} + 2$, so we use $x = 10$ to solve for c. So, $10 = \frac{1}{c} + 2$. This gives us $c = \frac{1}{8}$.

(c) For the point to lie on the x-axis, $y = 0$. The y-coordinate has the form 2+2c, so we use $y = 0$ to solve for c. So, $0 = 2 + 2c$. This gives us $c = -1$.

2.3 Solutions to Exercises

1. (a) 696   (b) 4 years

(c) $696/4 = 174$ students/year   (d) $1001 - 4(174) = 305$

(e) $P(t) = 174t + 305$ (f) $P(11) = 174(11) + 305 = 2219$ people

3. (a) From this information we can make two ordered pairs (410, 71.50) and (720,118). The slope between these two points is $\frac{118-71.50}{720-410} = 0.15$. So, the equation of the line has the form $f(x) = 0.15x + b$. To find b, we can substitute either of the two points in for x and $f(x)$ and solve for b. For this equation, $b = 10$. So, the final equation is $f(x) = 0.15x + 10$.

(b) The slope (0.15) is the price per minute of 15 cents, and the y-intercept (10) is

the flat monthly fee of ten dollars.

(c) $f(687) = 0.15(687) + 10 = 113.05$.

5. (a) From this information we can make two ordered pairs (1991, 4360) and (1999,5880). The slope between these two points is $\frac{5880-4360}{1999-1991} = 190$. The slope represents population growth of moose per year. If we want the equation to represent population growth in years after 1990, we have to figure out the moose population in 1990, which will be the y-intercept. We can subtract 190 from the population in 1991 to get the population in 1990, So $b = 4170$. So, the final equation is $f(t) = 190t + 4170$.

(b) 2003 is 13 years after 1990, so we will evaluate $f(13) = 6640$.

7. (a) From this information we have an ordered pair (16,2010) and the slope of -2.1. The slope is negative because the helium is being depleted. The slope represents the amount of helium being depleted each year. We want the equation to represent helium reserves in terms of the number of years since 2010, so the y-intercept will be helium amount in 2010, which is given to be 16. So, the final equation is $R(t) = -2.1t + 16$.

(b) 2015 is 5 years after 2010, so we will evaluate $R(5) = 5.5$.

(c) We want to know the value of t when $R(t) = 0$. So, we replace $R(t)$ with 0 and solve for t. $0 = -2.1t + 16$, so $t = 7.6$ years.

9. The two cell phone plans can be modeled by two linear equations: the first plan by $y = 0.26x$, the second plan by $y = 19.95 + 0.11x$. When the two equations are equal to each other, the cost for that amount of minutes will be the same for both plans. To find that point, we can set the equations equal to each other and solve for x. So, $0.26x = 19.95 + 0.11x$. This gives us $x = 133$. So, if you use less than 133 minutes, the first plan is cheaper, if you use more than 133 minutes, the second plan is cheaper.

11. The two pay options can be modeled by two linear equations: the first by $f(s) = 17,000 + 0.12s$, the second by $g(s) = 20,000 + 0.05s$. When the two equations are equal to each other, the income for that amount of sales will be the same for both options.

To find that point, we can set the equations equal to each other and solve for x. So, $17{,}000 + 0.12s = 20{,}000 + 0.05s$. This gives us $s = 42{,}857$. So, if you sell less than \$42,857 of jewelry, option A produces a larger income, if you sell more than \$42,857 of jewelry, option B produces a larger income.

13. It is useful to draw a picture for this problem (see below). The area of a triangle is found by $A = \frac{1}{2}bh$. We know that the

base$(b) = 9$, but we don't know the height(h). We do know, however, that the height of the triangle will be the x value at the point where the two lines intersect. The equation for the line perpendicular to $f(x)$ is $g(x) = \frac{7}{6}x$, because its slope will be the opposite reciprocal and it has a y-intercept at (0,0). To find the point where the two lines intersect, we can set them equal to each other and solve for x. This gives us $x = 4.44$, which is the height of the triangle. So, the Area $= \frac{1}{2}(9)(4.44) = 19.98$.
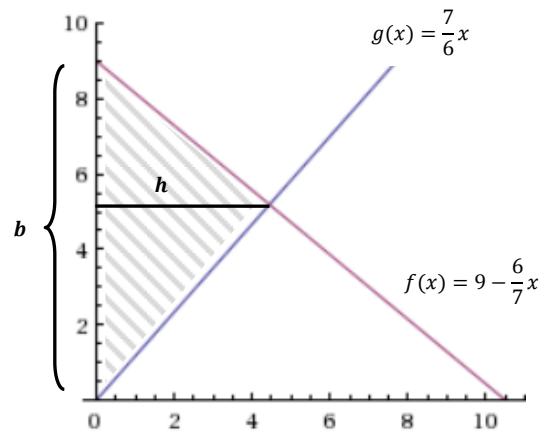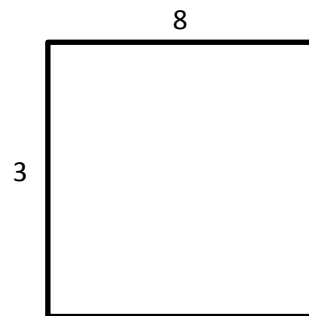
15. The equation of the line parallel to $f(x)=1+2x$ which passes through (2, 7) has equation $y-y_1=m(x-x_1)$ with $m=2$, $x_1=2$, and $y_1=7$:
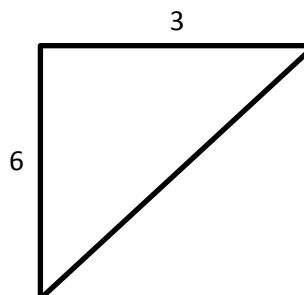
$y-7=2(x-2) \Rightarrow y=g(x)=2x+3$.
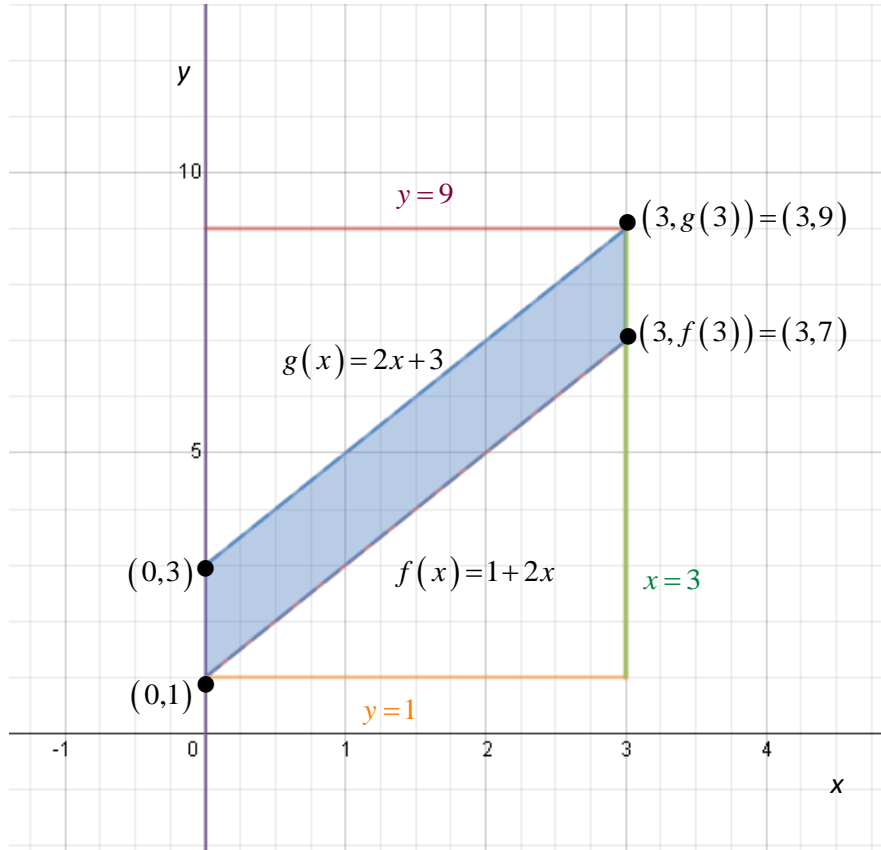
The area of the rectangle drawn around the

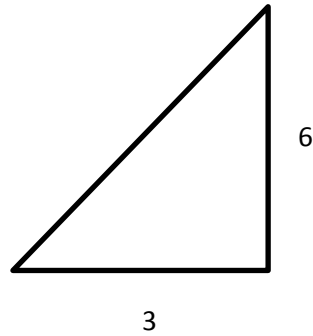parallelogram is $(9-1)\times(3-0)=24$ square units.

The area of the upper triangle is

$\frac{1}{2}(9-3)(3-0)=\frac{1}{2}\times6\times3=9$ square units.

The area of the lower triangle is

$$\frac{1}{2}(7-1)(3-0)=\frac{1}{2}\times 6\times 3=9$$ square units.


6

3

Therefore, the area of the region shaded in blue in the original figure -- the area of the parallelogram bounded by the y-axis, the line $x=3$, the line $f(x)=1+2x$, and the line parallel to $f(x)$ passing through (2, 7) which has equation $g(x)=2x+3$ -- is the area of the rectangle minus the area of the two triangles. So, the area of the parallelogram is $24-9-9=6$ square units.

Answer: 6

<u>Note</u>: there are other ways to solve this problem, such as using the formula for the area of a rectangle: $A = bh$. Imagining turning the parallelogram so that the segment along the y-axis is the base. The solution would start the same way as the solution above to show that $b = 2$. Looking at the figure this way, the height to the top is 3, so we see that we get the same answer as above: $A = bh = 2 \cdot 3 = 6$.

17. The area of the triangle is $A = \frac{1}{2}bh$. From the figure on the right we can see that the height is b, and the value of the x-intercept of the line $f(x)$. The x-intercept is found by replacing $f(x)$ with 0 and solving for x. This gives $x = -\frac{b}{m}$. So, the area in terms of m and b is $A = \left(\frac{1}{2}\right)\left(-\frac{b}{m}\right)(b)$ which can be simplified to $A = -\frac{b^2}{2m}$.

$(0, b)$

$f(x) = mx + b$

$\left(-\frac{b}{m}, 0\right)$
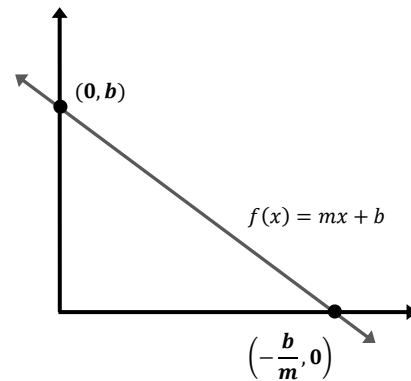
19. (a) Mississippi home values increased at a rate of $\frac{71,400-25,200}{2000-1950} = 924$ dollars/year,

and the Hawaii home values increased at a rate of $\frac{272,700-74,400}{2000-1950} = 3,966$ dollars/year.

So, Hawaii's home values increased faster.

   (b) 80,640

   (c) We can model these two equations in the following way: $M(x) = 71,400 + 924x$ and

   $H(x) = 272,700 + 3,966x$, where x is the number of years after 2000. To find

   when house values are the same in both states, we can set the equations equal to

   each other and solve for x. This gives $x = -66.17$, which would mean 66 years

   before 2000, so 1934.

21. We can think of these points on the coordinate plane as shown in the figure to the right. Pam will be closest to Paris when there is the shortest distance between her and Paris, which is when the dotted line is perpendicular to her path. We can find the equations of both lines, if we make her starting place R(0,0) and her
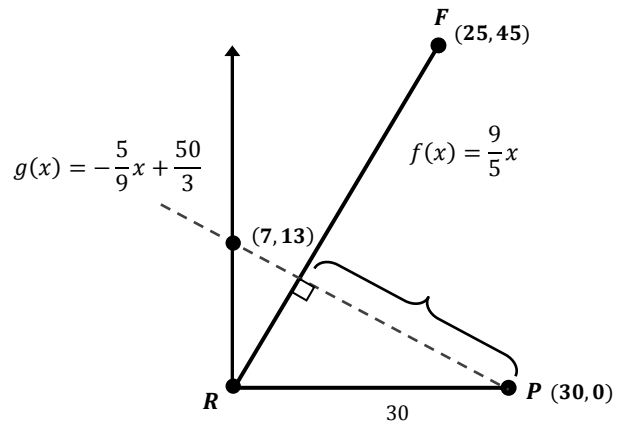

ending place F(25,45). The equation for her path is $f(x) = \frac{9}{5}x$, and the equation for the

dotted line is perpendicular to her path, so it has a slope of $-\frac{5}{9}$, and we know the point

(30,0) lies on this line, so we can find the y-intercept to be $\left(0, \frac{50}{3}\right)$. The equation for the

dotted line is $g(x) = -\frac{5}{9}x + \frac{50}{3}$. Knowing these equations helps us to find the point

where they intersect, by setting them equal to each other and solving for x. This point is

(7.09, 12.76), which on the graph is rounded to (7,13). So, when she is at this point she is
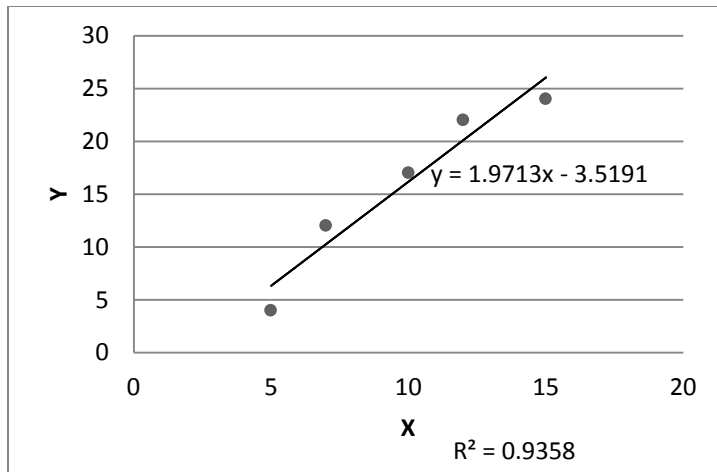
closest to Paris, and we can calculate this distance using the distance formula with points (7,13) and (30,0), which comes out to approximately 26.4 miles.
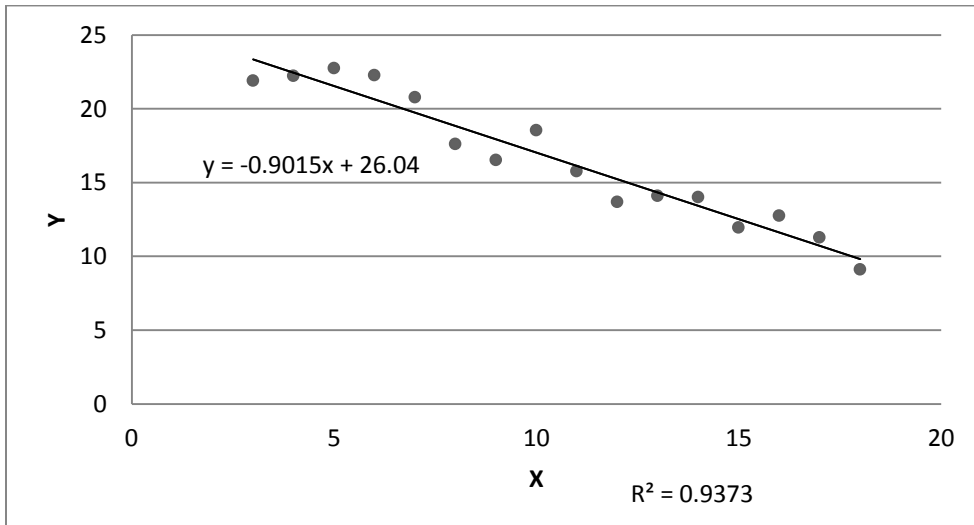
2.4 Solutions to Exercises

1.
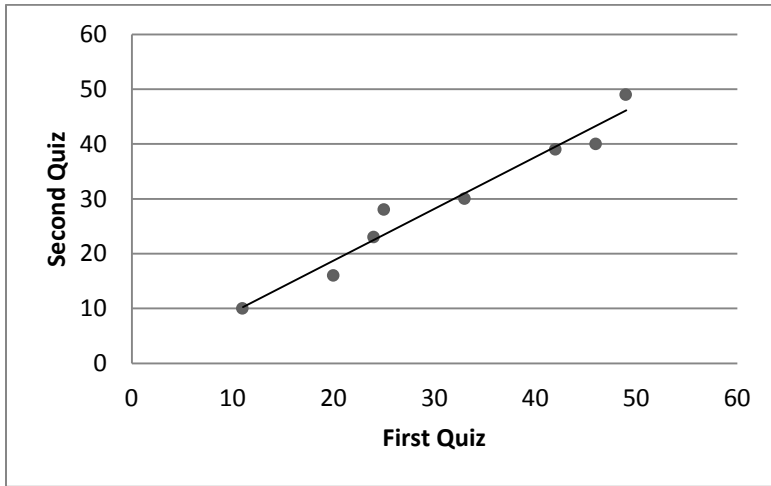
3.

Regression line equation: $f(x) = 1.971x - 3.519$.

Correlation coefficient: R=0.966954.

5.

Regression line equation: $f(x) = -0.901x + 26.04$.

Correlation coefficient, R=-0.967988

7. With the equation of a line $y = ax + b$ we are given $a = -1.341$ and $b = 32.234$, let our regression line be, $y = -1.341x + 32.234$. Since our correlation coefficient is close to negative one ($r = -0.896$) we know that the regression line will be a relatively good fit for the data and thus will give us a good prediction. Since x is the number of hours someone watches TV and y is the amount of sit-ups someone can do we can plug in 11 for x in our equation to get the predicted amount of sit-ups that person can do. That is,

$$y = -1.341(11) + 32.234 = \frac{17483}{1000} \approx 17.483.$$

A person who watches 11 hours of TV a day can do a predicted 17.5 sit-ups.

9. Noticing that r is positive and close to one, we look for a scatter plot that is increasing and that has plotted points that are close to the line of regression. D. $r = 0.95$

11. Noticing that r is positive but that r is not close to one, we look for a scatter plot that is increasing and has points that are further away from the regression line. A. r = 0.26

13.
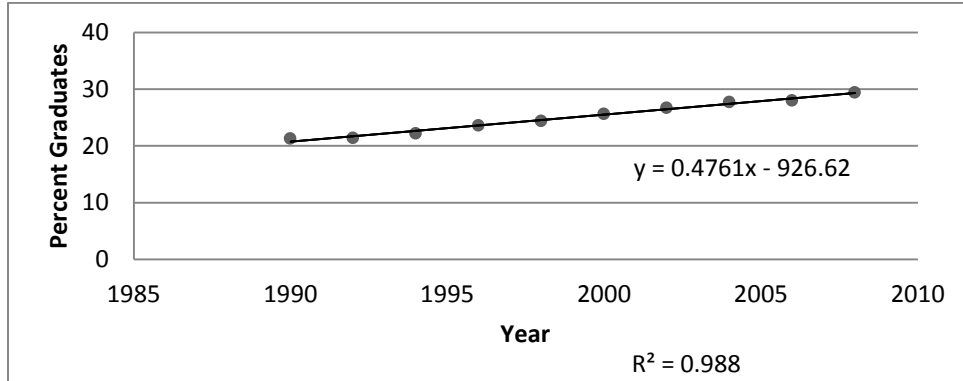
From graphing the data it is apparent that the trend appears linear. Finding the equation for the regression line with a computer we can calculate what year (if the trend continues) the percentage will exceed 35%. That is $y = 0.476x - 926.6$, with x equal to the year and y equal to percentage of persons 25 years or older who are college graduates. We can plug in $y = 35$, so $35 = 0.476x - 926.6$; adding 926.6 to each side, $961.6 = 0.476x$. Then by dividing both sides by 0.476 we arrive at $x = 2020.17 \approx 2020$. And so we can conclude that if the trend continues we will arrive at 35% of college graduates being persons over 25 years of age in the year 2020.
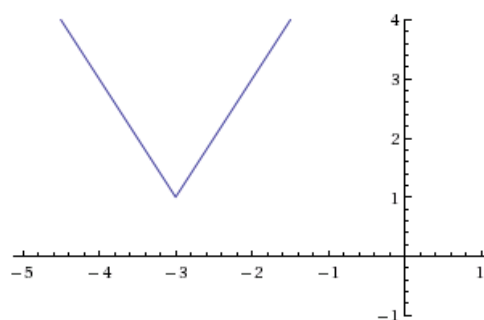
2.5 Solutions to Exercises

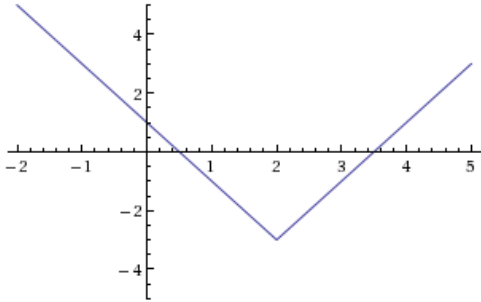1. $f(x) = \frac{1}{2}|x + 2| + 1$

3. $f(x) = -3.5|x - 3| + 3$

5.
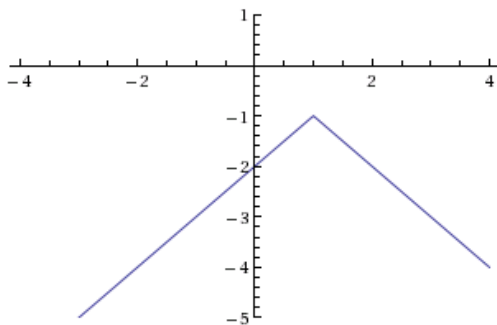
7.

9.

11. The only two numbers whose absolute value is 11 are -11 and 11, so:

$5x - 2 = 11$ or $5x - 2 = -11$

$5x = 13$ or $5x = -9$

$x = \frac{13}{5}$ or $x = -\frac{9}{5}$

13. $2|4 - x| = 7$

$|4 - x| = \frac{7}{2}$

$4 - x = \frac{7}{2}$ or $4 - x = -\frac{7}{2}$

$-x = -\frac{1}{2}$ or $-x = -\frac{15}{2}$

$x = \frac{1}{2}$ or $x = \frac{15}{2}$

15. $3|x + 1| - 4 = -2$

$|x + 1| = \frac{2}{3}$

$x + 1 = \frac{2}{3}$ or $x + 1 = -\frac{2}{3}$

$x = -\frac{1}{3}$ or $x = -\frac{5}{3}$

17.   Horizontal intercepts occur when $f(x) = 0$, giving the equation:

$2|x + 1| - 10 = 0$

Then: $|x + 1| = 5$

$\quad\quad x + 1 = 5 \quad$ or $\quad x + 1 = -5$

$\quad\quad x = 4 \quad\quad\quad$ or $\quad x = -6 \quad$ so the horizontal intercepts are (4, 0) and (-6, 0).

Vertical intercepts occur when $x = 0$: $f(0) = 2|0 + 1| - 10 = 2(1) - 10 = -8$. So the vertical intercept is (0, 8).

19. Horizontal intercepts occur when $f(x) = 0$, giving the equation:

$\quad\quad -3|x - 2| - 1 = 0$

Then: $|x - 2| = -\frac{1}{3}$. The absolute value of a quantity cannot give a negative value, so this equation has no solutions; there are no horizontal intercepts.

Vertical intercepts occur when $x = 0$: $f(0) = -3|0 - 2| - 1 = -3(2) - 1 = -7$. So the vertical intercept is (0, -7).

21. First, we solve the equation $|x + 5| = 6$:

$\quad\quad x + 5 = 6 \quad$ or $\quad x + 5 = -6$

$\quad\quad x = 1 \quad\quad\quad$ or $\quad x = -11$

From here, either use test points in the regions $x < -11$, $-11 < x < 1$, and $x > 1$ to determine which of these regions are solutions, or consider the graph of $f(x) = |x + 5|$. Using the prior method, let's test the points with *x*-coordinates -12, 0, and 2 (though you could use different numbers, as long as there's one from each of the three regions above):

$\quad\quad |-12 + 5| = |-7| = 7$, which is greater than 6, and thus not a solution to the original inequality $|x + 5| < 6$.

$\quad\quad |0 + 5| = |5| = 5$, which is less than 6, and thus a solution to $|x + 5| < 6$.

$\quad\quad |2 + 5| = |7| = 7$, which is greater than 6, and thus not a solution to $|x + 5| < 6$.

Since 0 is the only one that gave a solution to the inequality, the region it represents, $-11 < x < 1$, is the solution set.

23. First, we solve the equation $|x - 2| = 3$:

$\quad\quad x - 2 = 3 \quad$ or $\quad x - 2 = -3$

$\quad\quad x = 5 \quad\quad\quad$ or $\quad x = -1$

From here, either use test points in the regions $x < -1$, $-1 < x < 5$, and $x > 5$ to determine which of these regions are solutions, or consider the graph of $f(x) = |x - 2|$. Using the prior method, let's test the points with x-coordinates -2, 0, and 6:

$|-2 - 2| = |-4| = 4$, which is greater than 3, and thus a solution to $|x - 2| \geq 3$.

$|0 - 2| = |-2| = 2$, which is less than 3, and thus not a solution to $|x - 2| \geq 3$.

$|6 - 2| = |4| = 4$, which is greater than 3, and thus a solution to $|x - 2| \geq 3$.

Since -2 and 6 gave solutions to the inequality, the regions it represents give us the full solution set: $x \leq -1$ or $x \geq 5$.

25. First, we solve the equation $|3x + 9| = 4$:

$3x + 9 = 4 \quad$ or $\quad 3x + 9 = -4$

$x = -\dfrac{5}{3} \qquad$ or $\quad x = -\dfrac{13}{3}$

From here, either use test points in the regions $x < -\frac{13}{3}$, $-\frac{13}{3} < x < -\frac{5}{3}$, and $x > -\frac{5}{3}$ to determine which of these regions are solutions, or consider the graph of $f(x) = |3x + 9|$. Using the prior method, let's test the points with x-coordinates -5, -2, and 0:

$|3(-5) + 9| = |-6| = 6$, which is greater than 4, and thus not a solution to $|3x + 9| < 4$.

$|3(-2) + 9| = |3| = 3$, which is less than 4, and thus a solution to $|3x + 9| < 4$.

$|3(0) + 9| = |9| = 9$, which is greater than 4, and thus not a solution to $|3x + 9| < 4$.

Since -2 is the only one that gave a solution to the inequality, the region it represents, $-\dfrac{13}{3} < x < -\dfrac{5}{3}$, is the solution set.