Solutions to Selected Exercises

Voting Theory

1.

Number of voters	3	3	1	3	2
1st choice	A	A	B	B	C
2nd choice	B	C	A	C	A
3rd choice	C	B	C	A	B

3. a. 9+19+11+8 = 47
   b. 24 for majority; 17 for plurality
   c. Atlanta, with 19 first-choice votes
   d. Atlanta 94, Buffalo 111, Chicago 77. Winner: Buffalo
   e. Chicago eliminated, 11 votes go to Buffalo. Winner: Buffalo
   f. A vs B: B. A vs C: A. B vs C: B. B gets 2 pts, A 1 pt. Buffalo wins.

5. a. 120+50+40+90+60+100 = 460
   b. 231 for majority; 116 for plurality
   c. A with 150 first choice votes
   d. A 1140, B 1060, C 1160, D 1240. Winner: D
   e. B eliminated, votes to C. D eliminated, votes to A. Winner: A
   f. A vs B: B. A vs C: A. A vs D: D. B vs C: C. B vs D: D. C vs D: C
      A 1pt, B 1pt, C 2pt, D 2pt. Tie between C and D.
      Winner would probably be C since C was preferred over D

7. a. 33
   b. 17

9. Yes, B

11. B, with 17 approvals

13. Independence of Irrelevant Alternatives Criterion

15. Condorcet Criterion

Weighted Voting

1. a. 9 players
   b. 10+9+9+5+4+4+3+2+2 = 48
   c. 47

3. a. 9, a majority of votes
   b. 17, the total number of votes
   c. 12, which is 2/3 of 17, rounded up

5. a. P1 is a dictator (can reach quota by themselves)
   b. P1, since dictators also have veto power
   c. P2, P3, P4

7. a. none
   b. P1
   c. none

9. a. 11+7+2 = 20
   b. P1 and P2 are critical

11. Winning coalitions, with critical players underlined:
   {P1,P2} {P1,P2,P3} {P1,P2,P4} {P1,P2,P3,P4} {P1,P3} {P1,P3,P4}
   P1: 6 times, P2: 2 times, P3: 2 times, P4: 0 times. Total: 10 times
   Power: P1: 6/10 = 60%, P2: 2/10 = 20%, P3: 2/10 = 20%, P4: 0/10 = 0%

13. a. {P1} {P1,P2} {P1,P3} {P1,P4} {P1,P2,P3} {P1,P2,P4} {P1,P3,P4} {P1,P2,P3,P4}
   P1: 100%, P2: 0%, P3: 0%, P4: 0%
   b. {P1,P2} {P1,P3} {P1,P4} {P1,P2,P3} {P1,P2,P4} {P1,P3,P4} {P1,P2,P3,P4}
   P1: 7/10 = 70%, P2: 1/10 = 10%, P3: 1/10 = 10%, P4: 1/10 = 10%
   c. {P1,P2} {P1,P3} {P1,P2,P3} {P1,P2,P4} {P1,P3,P4} {P1,P2,P3,P4}
   P1: 6/10 = 60%, P2: 2/10 = 20%, P3: 2/10 = 20%, P4: 0/10 = 0%

15. P3 = 5. P3+P2 = 14. P3+P2+P1 = 27, reaching quota. P1 is critical.

17. Sequential coalitions with pivotal player underlined
   <P1,P2,P3> <P1,P3,P2> <P2,P1,P3> <P2,P3,P1> <P3,P1,P2> <P3,P2,P1>
   P1: 2/6 = 33.3%, P2: 2/6 = 33.3%, P3: 2/6 = 33.3%

19. a. 6, 7
   b. 8, given P1 veto power
   c. 9, given P1 and P2 veto power

21. If adding a player to a coalition could cause it to reach quota, that player would also be critical in that coalition, which means they are not a dummy. So a dummy cannot be pivotal.

23. We know P2+P3 can't reach quota, or else P1 wouldn't have veto power.
P1 can't reach quota alone.
P1+P2 and P1+P3 must reach quota or else P2/P3 would be dummy.
a. {P1,P2} {P1,P3} {P1,P2,P3}. P1: 3/5, P2: 1/5, P3: 1/5
b. <P1,P2,P3> <P1,P3,P2> <P2,P1,P3> <P2,P3,P1> <P3,P1,P2> <P3,P2,P1>
P1: 4/6, P2: 1/6, P3: 1/6

25. [4: 2, 1, 1, 1] is one of many possibilities

27. [56: 30, 30, 20, 20, 10]

29. [54: 10, 10, 10, 10, 10, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] is one of many possibilities

Fair Division

1. Chance values the veggie half at \$7.50 and pepperoni half at \$2.50.
A full pepperoni slice is ¼ of the pepperoni half. Value \$2.50/4 = \$0.625
A full veggie slice is ¼ of the veggie half. Value \$7.50/4 = \$1.875
A slice that is ½ pepperoni ½ veggie is value \$0.3125+\$0.9375 = \$1.25

3. Erin: Bowl 1, Catherine: Bowl 2, Shannon: Bowl 3

5. a. 25 Snickers @ \$0.01 each, 20 Milky Ways @ \$0.05 each, 60 Reese's @ \$0.02 each
Value: \$0.25 + \$1.00 + \$1.20 = \$2.45
b. No. Dustin values the whole bag at \$8, so a fair share would be \$4.
c. Lots of possibilities. Here's a couple:
80 Milky Ways, 0 Snickers, 0 Reese's
50 Snickers, 50 Milky Ways, 50 Reese's

7. a. Zoe
b. Maggie: s2, s3. Meredith: s1, s2. Holly: s3
c. Maggie: s2, Meredith: s1, Holly: s3, Zoe: s4

9. a. P5
b. \$6.50 (doesn't need to trim it much since they're last)
c. P4 would receive it, with value \$6.00 (since P4 would trim it)

11. a. (320+220)/4 = \$135
b. Desk and Vanity both go to A. A pays \$320 + \$220 - \$135 = \$405 to estate
B gets \$95, C gets \$125, D gets \$110.
c. Surplus of \$405 - \$95 - \$125 - \$110 = \$75 gets split, \$18.75 each.
A gets desk and vanity, pays \$386.25 to estate
B gets \$113.75, C gets \$143.75, D gets \$128.75

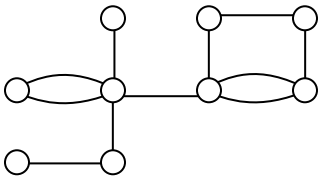
13. Fair shares: Abby: 10.333, Ben: 9, Carla: 7.667
Motorcycle to Abby, Car to Ben, Tractor to Abby, Boat to Abby
Initial: Abby pays $10.667, Ben pays $2, Carla gets $7.667
Surplus: $5; $1.667 each
Final: Abby gets Motorcycle, Tractor and Boat, pays $9
Ben gets Car, pays $0.333
Carla gets $9.334

15. Fair shares: Sasha: $135, Megan: $140
Sasha gets: Couch, detail cleaning. Value $80
Megan gets: TV, Stereo, carpets. Value: $260
Initial: Sasha gets $55, Megan pays $120.
Surplus: $65; $32.50 each
Final: Sasha gets Couch and does detail cleaning, gets $87.50
Megan gets TV and stereo, and cleans carpets, pays $87.50

17. a. s3, worth $270
b. s1 and s4 have combined value $440 for Greedy, so piece would be worth $220

Graph Theory

1.



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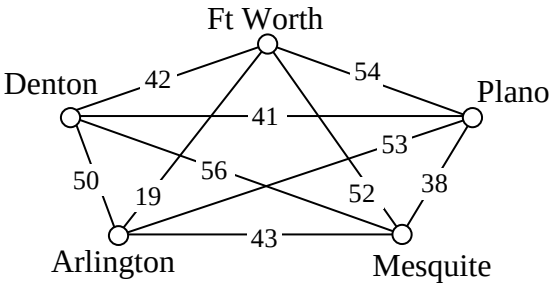

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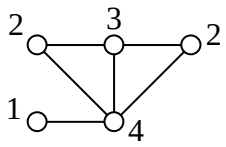
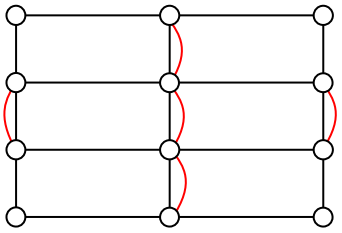


7. The first and the third graphs are connected

9. Bern to Frankfurt to Munchen to Berlin: 12hrs 50 min. (Though trip through Lyon, Paris and Amsterdam only adds 30 minutes)

11. The first graph has an Euler circuit. The last two graphs each have two vertices with odd degree.

13. One of several possible eulerizations requiring 5 duplications:

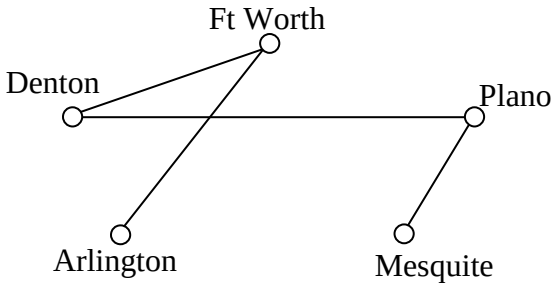


17. Only the middle graph has a Hamiltonian circuit.

19. a. Ft Worth, Arlington, Mesquite, Plano, Denton, Ft Worth: 183 miles
b. Same as part a
c. Same as part a

21. a. ABDCEA
b. ACEBDA
c. ADBCEA

23.



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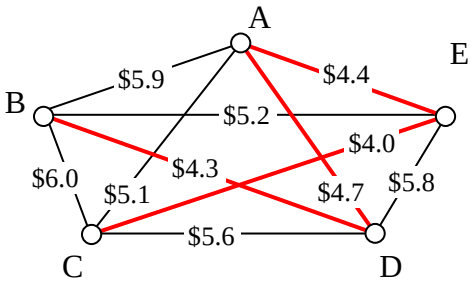
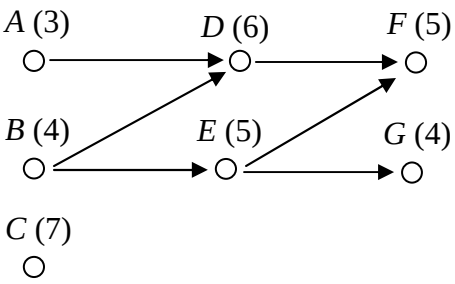


Scheduling

1.



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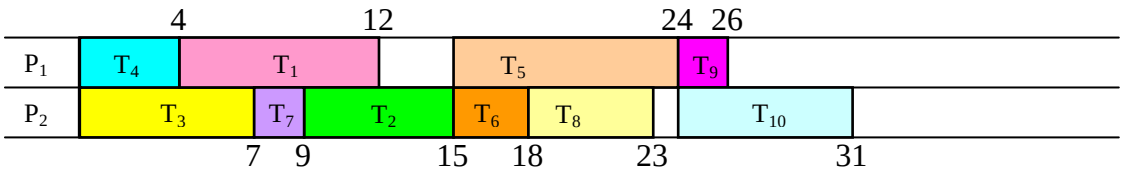

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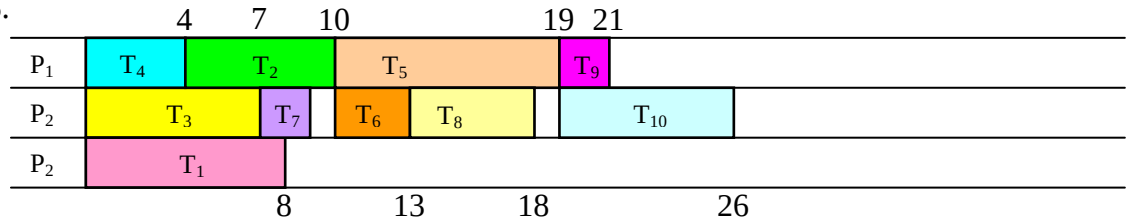
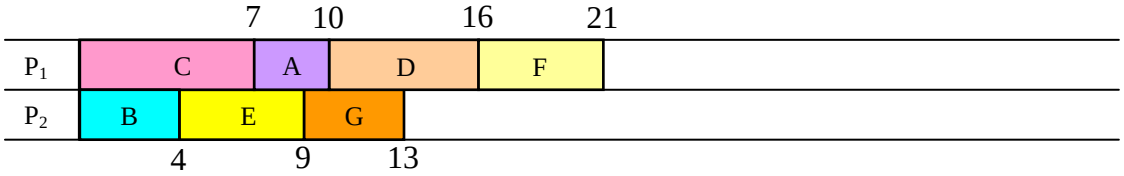
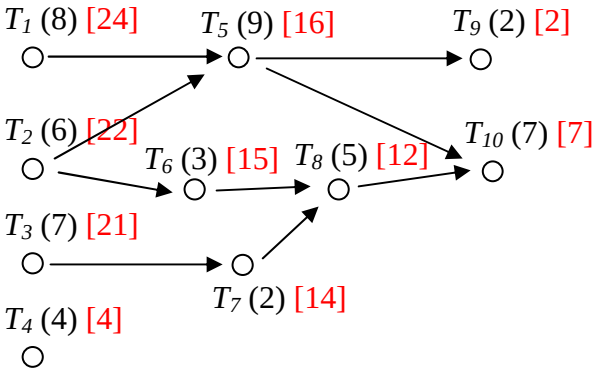


7. Priority List: T_5, T_1, T_3, T_{10}, T_2, T_8, T_4, T_6, T_7, T_9

9. Priority List: C, D, E, F, B, G, A

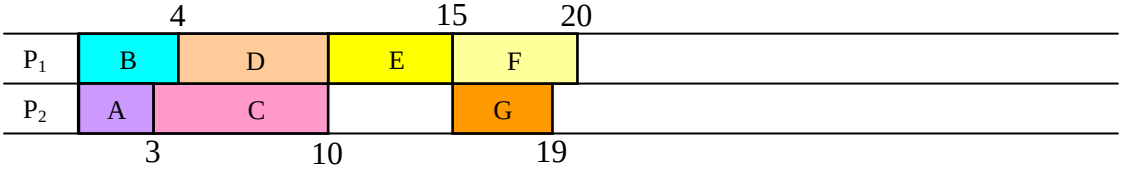


11. a.



b. Critical path: T1, T5, T10. Minimum completion time: 24
c. Critical path priority list: T_1, T_2, T_3, T_5, T_6, T_7, T_8, T_{10}, T_4, T_9

13. Critical path priority list: B, A, D, E, C, F, G



Growth Models

1. a. $P_0 = 20$. $P_n = P_{n-1} + 5$
   b. $P_n = 20 + 5n$

3. a. $P_1 = P_0 + 15 = 40+15 = 55$. $P_2 = 55+15 = 70$
   b. $P_n = 40 + 15n$
   c. $P_{10} = 40 + 15(10) = 190$ thousand dollars

5. Grew 64 in 8 weeks: 8 per week
   a. $P_n = 3 + 8n$
   b. $187 = 3 + 8n$. $n = 23$ weeks

7. a. $P_0 = 200$ (thousand), $P_n = (1+.09)\ P_{n-1}$ where *n* is years after 2000
   b. $P_n = 200(1.09)^n$
   c. $P_{16} = 200(1.09)^{16} = 794.061$ (thousand) = 794,061

9. Let n=0 be 1983. $P_n = 1700(2.9)^n$. 2005 is n=22. $P_{22} = 1700(2.9)^{22} = 25{,}304{,}914{,}552{,}324$ people. Clearly not realistic, but mathematically accurate.

11. If n is in hours, better to start with the explicit form. $P_0 = 300$. $P_4 = 500 = 300(1+r)^4$
    $500/300 = (1+r)^4$. $1+r = 1.136$. $r = 0.136$
    a. $P_0 = 300$. $P_n = (1.136)P_{n-1}$
    b. $P_n = 300(1.136)^n$
    c. $P_{24} = 300(1.136)^{24} = 6400$ bacteria

13. a. $P_0 = 100$ $P_n = P_{n-1} + 0.70\ (1 - P_{n-1} / 2000)\ P_{n-1}$
    b. $P_1 = 100 + 0.70(1 - 100/2000)(100) = 166.5$
    c. $P_2 = 166.5 + 0.70(1 - 166.5/2000)(166.5) = 273.3$

15. To find the growth rate, suppose n=0 was 1968. Then P_0 would be 1.60 and $P_8 = 2.30 = 1.60(1+r)^8$, $r = 0.0464$. Since we want n=0 to correspond to 1960, then we don't know P_0, but P_8 would $1.60 = P_0(1.0464)^8$. $P_0 = 1.113$.
    a. $P_n = 1.113(1.0464)^n$
    b. P_0= \$1.113, or about \$1.11
    c. 1996 would be n=36. $P_{36} = 1.113(1.0464)^{36} =$ \$5.697. Actual is slightly lower.

17. The population in the town was 4000 in 2005, and is growing by 4% per year.

Finance

1. $A = 200 + .05(200) =$ \$210

3. I=200. t = 13/52 (13 weeks out of 52 in a year). $P_0 = 9800$
   $200 = 9800(r)(13/52)$ $r = 0.0816 = 8.16\%$ annual rate

5. $P_{10} = 300(1+.05/1)^{10(1)}$ = \$488.67

7. a. $P_{20} = 2000(1+.03/12)^{20(12)}$ = \$3641.51 in 20 years
   b. 3641.51 – 2000 = \$1641.51 in interest

9. $P_8 = P_0(1+.06/12)^{8(12)} = 6000$. P_0 = \$3717.14 would be needed

11. a. $P_{30} = \dfrac{200\left((1+0.03/12)^{30(12)}-1\right)}{0.03/12}$ = \$116,547.38
    b. 200(12)(30) = \$72,000
    c. \$116,547.40 - \$72,000 = \$44,547.38 of interest

13. a. $P_{30} = 800{,}000 = \dfrac{d\left((1+0.06/12)^{30(12)}-1\right)}{0.06/12}$ d = \$796.40 each month
    b. \$796.40(12)(30) = \$286,704
    c. \$800,000 - \$286,704 = \$513,296 in interest

15. a. $P_0 = \dfrac{30000\left(1-(1+0.08/1)^{-25(1)}\right)}{0.08/1}$ = \$320,253.29
    b. 30000(25) = \$750,000
    c. \$750,000 - \$320,253.29 = \$429,756.71

17. $P_0 = 500{,}000 = \dfrac{d\left(1-(1+0.06/12)^{-20(12)}\right)}{0.06/12}$ d = \$3582.16 each month

19. a. $P_0 = \dfrac{700\left(1-(1+0.05/12)^{-30(12)}\right)}{0.05/12}$ = a \$130,397.13 loan
    b. 700(12)(30) = \$252,000
    c. \$252,200 - \$130,397.13 = \$121,602.87 in interest

21. $P_0 = 25{,}000 = \dfrac{d\left(1-(1+0.02/12)^{-48}\right)}{0.02/12}$ = \$542.38 a month

23. a. Down payment of 10% is \$20,000, leaving \$180,000 as the loan amount
    b. $P_0 = 180{,}000 = \dfrac{d\left(1-(1+0.05/12)^{-30(12)}\right)}{0.05/12}$ d = \$966.28 a month
    c. $P_0 = 180{,}000 = \dfrac{d\left(1-(1+0.06/12)^{-30(12)}\right)}{0.06/12}$ d = \$1079.19 a month

25. First 5 years: $P_5 = \dfrac{50\left((1+0.08/12)^{5(12)}-1\right)}{0.08/12}$ = \$3673.84

Next 25 years: $3673.84(1+.08/12)^{25(12)}$ = $26,966.65

27. Working backwards, $P_0 = \frac{10000\left(1-(1+0.08/4)^{-10(4)}\right)}{0.08/4}$ = $273,554.79 needed at retirement. To end up with that amount of money, $273{,}554.70 = \frac{d\left((1+0.08/4)^{15(4)}-1\right)}{0.08/4}$. He'll need to contribute d = $2398.52 a quarter.

Statistics

1. a. Population is the current representatives in the state's congress
   b. 106
   c. the 28 representatives surveyed
   d. 14 out of 28 = ½ = 50%
   e. We might expect 50% of the 106 representatives = 53 representatives

3. This suffers from leading question bias

5. This question would likely suffer from a perceived lack of anonymity

7. This suffers from leading question bias

9. Quantitative

11. Observational study

13. Stratified sample

15. a. Group 1, receiving the vaccine
    b. Group 2 is acting as a control group. They are not receiving the treatment (new vaccine).
    c. The study is at least blind. We are not provided enough information to determine if it is double-blind.
    d. This is a controlled experiment

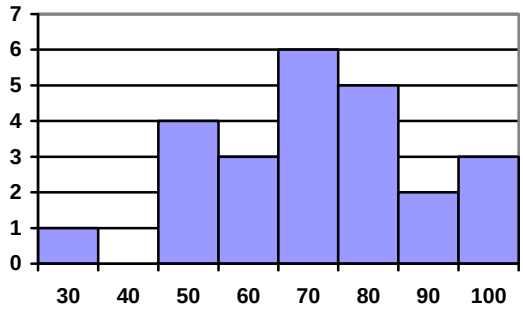
17. a. Census
    b. Observational study

Describing Data

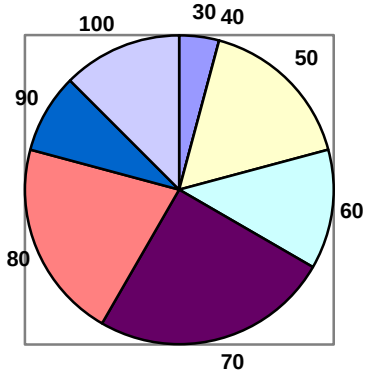
1. a. Different tables are possible

Score	Frequency
30	1
40	0
50	4
60	3
70	6
80	5
90	2
100	3

b. This is technically a bar graph, not a histogram:



c.



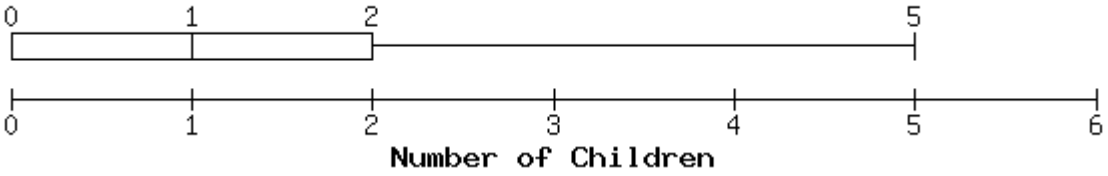
3. a. 5+3+4+2+1 = 15
b. 5/15 = 0.3333 = 33.33%

5. Bar is at 25%. 25% of 20 = 5 students earned an A

7. a. (7.25+8.25+9.00+8.00+7.25+7.50+8.00+7.00)/8 = $7.781
b. In order, 7.50 and 8.00 are in middle positions. Median = $7.75
c. 0.25*8 = 2. Q1 is average of 2nd and 3rd data values: $7.375
0.75*8 = 6. Q3 is average of 6th and 7th data values: $8.125
5-number summary: $7.00, $7.375, $7.75, $8.125, $9.00

9. a. (5*0 + 3*1 + 4*2 + 2*3 + 1*5)/15 = 1.4667
   b. Median is 8th data value: 1 child
   c. 0.25*15 = 3.75. Q1 is 4th data value: 0 children
      0.75*15 = 11.25. Q3 is 12th data value: 2 children
      5-number summary: 0, 0, 1, 2, 5

d.



11. Kendra makes $90,000. Kelsey makes $40,000. Kendra makes $50,000 more.

Historical Counting

1. Partial answer: Jars: 3 singles, 3 @ x2, 2 @ x6, 1 @ x12. 3+6+12+12 = 33

3. 113

5. 3022

7. 53

9. 1100100

11. 332

13. 111100010

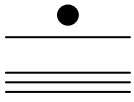
15. 7,1,10 base 12 = 1030 base 10

17. 6,4,2 base 12 = 914 base 10

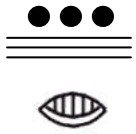
19. 175 base 10 = 1,2,7 base 12 = ♉♊♏

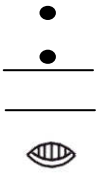
21. 10000 base 10 = 5,9,5,4 base 12 = ♍♑♍♌

23. 135 = 6,15 base 20 =



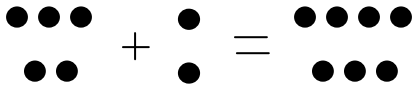
25. 360 = 18,0 base 20 =



27. 10500 = 1,6,5,0 base 20 

29. 1,2,12 base 20 = 452 base 10

31. 3,0,3 base 20 = 1203 base 10

33. 

35.