

Finance

We have to work with money every day. While balancing your checkbook or calculating your monthly expenditures on espresso requires only arithmetic, when we start saving, planning for retirement, or need a loan, we need more mathematics.

Simple Interest

Discussing interest starts with the **principal**, or amount your account starts with. This could be a starting investment, or the starting amount of a loan. Interest, in its most simple form, is calculated as a percent of the principal. For example, if you borrowed \$100 from a friend and agree to repay it with 5% interest, then the amount of interest you would pay would just be 5% of 100: $\$100(0.05) = \5 . The total amount you would repay would be \$105, the original principal plus the interest.

Simple One-time Interest

$$I = P_0 r$$

$$A = P_0 + I = P_0 + P_0 r = P_0(1 + r)$$

I is the interest

A is the end amount: principal plus interest

P_0 is the principal (starting amount)

r is the interest rate (in decimal form. Example: 5% = 0.05)

Example: A friend asks to borrow \$300 and agrees to repay it in 30 days with 3% interest. How much interest will you earn?

$P_0 = \$300$ (the principal)

$r = 0.03$ (3% rate)

$I = \$300(0.03) = \9 . You will earn \$9 interest.

One-time simple interest is only common for extremely short-term loans. For longer term loans, it is common for interest to be paid on a daily, monthly, quarterly, or annual basis. In that case, interest would be earned regularly. For example, bonds are essentially a loan made to the bond issuer (a company or government) by you, the bond holder. In return for the loan, the issuer agrees to pay interest, often annually. Bonds have a maturity date, at which time the issuer pays back the original bond value.

Example: Suppose your city is building a new park, and issues bonds to raise the money to build it. You obtain a \$1,000 bond that pays 5% interest annually that matures in 5 years. Each year, you would earn 5% interest: $\$1000(0.05) = \50 in interest. So over the course of five years, you would earn a total of \$250 in interest. When the bond matures, you would receive back the \$1,000 you originally paid, leaving you with a total of \$1,250.

Simple Interest over Time

$$I = P_0rt$$

$$A = P_0 + I = P_0 + P_0rt = P_0(1 + rt)$$

I is the interest

A is the end amount: principal plus interest

P_0 is the principal (starting amount)

r is the interest rate (in decimal form. Example: 5% = 0.05)

t is time

Example: Treasury Notes (T-notes) are bonds issued by the federal government to cover its expenses. Suppose you obtain a \$1,000 T-note with a 4% annual rate, paid semi-annually, with a maturity in 4 years. How much interest will you earn?

First, it is important to know that interest rates are usually given as an **annual percentage rate (APR)** – the total interest that will be paid in the year. Since interest is being paid semi-annually (twice a year), the 4% interest will be divided into two 2% payments.

$P_0 = \$1000$ (the principal)

$r = 0.02$ (2% rate per half-year)

$t = 8$ (4 years = 8 half-years)

$I = \$1000(0.02)(8) = \160 . You will earn \$160 interest total over the four years.

Compound Interest

With simple interest, we were assuming that we pocketed the interest when we received it. In a standard bank account, any interest we earn is automatically added to our balance. This reinvestment of interest is called **compounding**.

Example: Suppose that we deposit \$1000 in a bank account offering 3% interest, compounded monthly. How will our money grow?

The 3% interest is an annual percentage rate (APR) – the total interest to be paid during the year. Since interest is being paid monthly, each month, we will earn $3\%/12 = 0.25\%$ per month.

So in the first month,

$P_0 = \$1000$

$r = 0.0025$ (0.25%)

$I = \$1000(0.0025) = \2.50

$A = \$1000 + \$2.50 = \$1002.50$

So in the first month, we will earn \$2.50 in interest, raising our account balance to \$1002.50. In the second month,

$$P_0 = \$1002.50$$

$$r = 0.0025 \text{ (0.25\%)}$$

$$I = \$1002.50 (0.0025) = \$2.51 \text{ (rounded)}$$

$$A = \$1002.50 + \$2.51 = \$1005.01$$

Notice that in the second month we earned more interest than we did in the first month. This is because we earned interest not only on the original \$1000 we deposited, but we also earned interest on the \$2.50 of interest we earned the first month. This is the key advantage that **compounding** of interest gives us.

Calculating out a few more months:

Month	Starting balance	Interest earned	Ending Balance
1	1000.00	2.50	1002.50
2	1002.50	2.51	1005.01
3	1005.01	2.51	1007.52
4	1007.52	2.52	1010.04
5	1010.04	2.53	1012.57
6	1012.57	2.53	1015.10
7	1015.10	2.54	1017.64
8	1017.64	2.54	1020.18
9	1020.18	2.55	1022.73
10	1022.73	2.56	1025.29
11	1025.29	2.56	1027.85
12	1027.85	2.57	1030.42

To find an equation to represent this, if P_m represents the amount of money after m months, then

$$P_0 = \$1000$$

$$P_m = (1+0.0025)P_{m-1}$$

To build an explicit equation for growth,

$$P_0 = \$1000$$

$$P_1 = 1.0025P_0 = 1.0025 (1000)$$

$$P_2 = 1.0025P_1 = 1.0025 (1.0025 (1000)) = 1.0025^2(1000)$$

$$P_3 = 1.0025P_2 = 1.0025 (1.0025^2(1000)) = 1.0025^3(1000)$$

$$P_4 = 1.0025P_3 = 1.0025 (1.0025^3(1000)) = 1.0025^4(1000)$$

Observing a pattern, we could conclude

$$P_m = (1.0025)^m(\$1000)$$

Notice that the \$1000 in the equation was P_0 , the starting amount. We found 1.0025 by adding one to the growth rate divided by 12, since we were compounding 12 times per year. Generalizing our result, we could write

$$P_m = P_0 \left(1 + \frac{r}{k} \right)^m$$

In this formula:

m is the number of compounding periods (months in our example)

r is the annual interest rate

k is the number of compounds per year.

While this formula works fine, it is more common to use a formula that involves the number of years, rather than the number of compounding periods. If N is the number of years, then $m = Nk$. Making this change gives us the standard formula for compound interest:

Compound Interest

$$P_N = P_0 \left(1 + \frac{r}{k} \right)^{Nk}$$

In this formula:

P_N is the balance in the account after N years.

P_0 is the starting balance of the account (also called initial deposit, or principal)

r is the annual interest rate (in decimal form. Example: 5% = 0.05)

k is the number of compounding periods in one year.

If the compounding is done annually (once a year), $k = 1$.

If the compounding is done quarterly, $k = 4$.

If the compounding is done monthly, $k = 12$.

If the compounding is done daily, $k = 365$.

The most important thing to remember about using this formula is that it assumes that we put money in the account once and let it sit there earning interest.

If you're unsure how to raise numbers to large powers, see the *Using Your Calculator* section at the end of the chapter.

Example 1. A certificate of deposit (CD) is savings instrument that many banks offer. It usually gives a higher interest rate, but you cannot access your investment for a specified length of time. Suppose you deposit \$3000 in a CD paying 6% interest, compounded monthly. How much will you have in the account after 20 years?

In this example,

$P_0 = \$3000$ (the initial deposit)

$r = 0.06$ (6%)

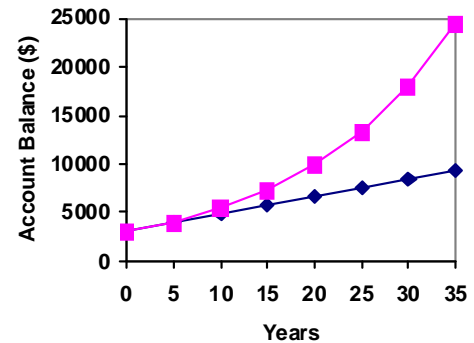
$k = 12$ (12 months in 1 year)

$N = 20$, since we're looking for P_{20}

$$\text{So } P_{20} = 3000 \left(1 + \frac{0.06}{12} \right)^{20 \times 12} = \$9930.61 \quad (\text{round your answer to the nearest penny})$$

Let us compare the amount of money you will have from compounding against the amount you would have just from simple interest

Years	Simple Interest (\$15 per month)	6% compounded monthly = 0.5% each month.
5	\$3900	\$4046.55
10	\$4800	\$5458.19
15	\$5700	\$7362.28
20	\$6600	\$9930.61
25	\$7500	\$13394.91
30	\$8400	\$18067.73
35	\$9300	\$24370.65



As you can see, over a long period of time, compounding makes a large difference in the account balance.

Example 2. You know that you will need \$40,000 for your child's education in 18 years. If your account earns 4% compounded quarterly, how much would you need to deposit now to reach your goal?

In this example,
 We're looking for P_0 .
 $r = 0.04$ (4%)
 $k = 4$ (4 quarters in 1 year)
 $N = 18$
 $P_{18} = \$40,000$

In this case, we're going to have to set up the equation, and solve for P_0 .

$$40000 = P_0 \left(1 + \frac{0.04}{4} \right)^{18 \times 4}$$

$$40000 = P_0 (2.0471)$$

$$P_0 = \frac{40000}{2.0471} = \$19539.84$$

So you would need to deposit \$19,539.84 now to have \$40,000 in 18 years.

Note on rounding: You may have to round numbers sometimes while working these problems. Try to keep 3 significant digits after the decimal place (keep 3 non-zero numbers).

For example:

Round 1.00563663 to 1.00564

Round 2.06127 to 2.0613

For more information on rounding, see the *Note on Rounding* at the end of the chapter.

Annuities

For most of us, we aren't able to just put a large sum of money in the bank today. Instead, we save by depositing a smaller amount of money from each paycheck into the bank. This idea is called a **savings annuity**. This is what most retirement plans are.

An annuity can be described recursively in a fairly simple way. Recall that basic compound interest follows from the relationship

$$P_m = \left(1 + \frac{r}{k}\right) P_{m-1}$$

For a savings annuity, we simply need to add a deposit, d , to the account with each compounding period:

$$P_m = \left(1 + \frac{r}{k}\right) P_{m-1} + d$$

Taking this equation from recursive form to explicit form is a bit trickier than with compound interest. It will be easiest to see by working with an example rather than working in general.

Example: Suppose we will deposit \$100 each month into an account paying 6% interest. We assume that the account is compounded with the same frequency as we make deposits unless stated otherwise. So in this example:

$$r = 0.06 \text{ (6\%)}$$

$$k = 12 \text{ (12 compounds/deposits per year)}$$

$$d = \$100 \text{ (our deposit per month)}$$

So

$$P_m = \left(1 + \frac{0.06}{12}\right) P_{m-1} + 100 = (1.005) P_{m-1} + 100$$

Assuming we start with an empty account, we can begin using this relationship:

$$P_0 = 0$$

$$P_1 = (1.005) P_0 + 100 = 100$$

$$P_2 = (1.005) P_1 + 100 = (1.005)(100) + 100 = 100(1.005) + 100$$

$$P_3 = (1.005) P_2 + 100 = (1.005)(100(1.005) + 100) + 100 = 100(1.005)^2 + 100(1.005) + 100$$

Continuing this pattern,

$$P_m = 100(1.005)^{m-1} + 100(1.005)^{m-2} + \cdots + 100(1.005) + 100$$

In other words, after m months, the first deposit will have earned compound interest for $m-1$ months. The second deposit will have earned interest for $m-2$ months. Last months deposit would have earned only one month worth of interest. The most recent deposit will have earned no interest yet.

This equation leaves a lot to be desired, though – it doesn't make calculating the ending balance any easier! To simplify things, multiply both sides of the equation by 1.005:

$$1.005P_m = 1.005\left(100(1.005)^{m-1} + 100(1.005)^{m-2} + \dots + 100(1.005) + 100\right)$$

Distributing on the right side of the equation gives

$$1.005P_m = 100(1.005)^m + 100(1.005)^{m-1} + \dots + 100(1.005)^2 + 100(1.005)$$

Now we'll line this up with like terms from our original equation, and subtract each side

$$\begin{aligned} 1.005P_m &= 100(1.005)^m + 100(1.005)^{m-1} + \dots + 100(1.005) \\ P_m &= 100(1.005)^{m-1} + \dots + 100(1.005) + 100 \end{aligned}$$

Almost all the terms cancel on the right hand side, leaving

$$1.005P_m - P_m = 100(1.005)^m - 100$$

Solving for P_m

$$\begin{aligned} 0.005P_m &= 100\left((1.005)^m - 1\right) \\ P_m &= \frac{100\left((1.005)^m - 1\right)}{0.005} \end{aligned}$$

Replacing m months with $12N$, where N is years gives

$$P_N = \frac{100\left((1.005)^{12N} - 1\right)}{0.005}$$

Recall 0.005 was r/k and 100 was the deposit d . 12 was k , the number of deposit each year. Generalizing this, we get:

Annuity Formula

$$P_N = \frac{d \left(\left(1 + \frac{r}{k} \right)^{Nk} - 1 \right)}{\left(\frac{r}{k} \right)}$$

In this formula:

P_N is the balance in the account after N years.

d is the regular deposit (the amount you deposit each year, each month, etc.)

r is the annual interest rate (in decimal form. Example: 5% = 0.05)

k is the number of compounding periods in one year.

The compounding frequency is not always explicitly given. But:

If you make your deposits every month, use monthly compounding, $k = 12$.

If you make your deposits every year, use yearly compounding, $k = 1$.

If you make your deposits every quarter, use quarterly compounding, $k = 4$.

Etc.

The most important thing to remember about using this formula is that it assumes that you put money in the account on a regular schedule (every month, year, quarter, etc.) and let it sit there earning interest.

Compound interest: One deposit

Annuity: Many deposits.

Example 1. A traditional individual retirement account (IRA) is a special type of retirement account in which the money you invest is exempt from income taxes until you withdraw it. If you deposit \$100 each month into an IRA earning 6% interest. How much will you have in the account after 20 years?

In this example,

$d = \$100$ (the monthly deposit)

$r = 0.06$ (6%)

$k = 12$ (since we're doing monthly deposits, we'll compound monthly)

$N = 20$, since we're looking for P_{20}

Putting this into the equation:

$$P_{20} = \frac{100 \left(\left(1 + \frac{0.06}{12} \right)^{20(12)} - 1 \right)}{\left(\frac{0.06}{12} \right)}$$

$$P_{20} = \frac{100 \left((1.005)^{240} - 1 \right)}{(0.005)}$$

$$P_{20} = \frac{100(3.310 - 1)}{(0.005)}$$

$$P_{20} = \frac{100(2.310)}{(0.005)} = \$46200$$

So you will have \$46,200 after 20 years. Notice that you deposited into the account a total of \$24,000 (\$100 a month for 240 months). The difference between what you end up with and how much you put in is the interest earned. In this case it is \$46,200 - \$24,000 = \$22,200.

Example 2. You want to have \$200,000 in your account when you retire in 30 years. Your retirement account earns 8% interest. How much do you need to deposit each month to meet your retirement goal?

In this example,

We're looking for d .

$r = 0.08$ (8%)

$k = 12$ (since we're depositing monthly)

$N = 30$ (30 years)

$P_{30} = \$200,000$

In this case, we're going to have to set up the equation, and solve for d .

$$200,000 = \frac{d \left(\left(1 + \frac{0.08}{12} \right)^{30(12)} - 1 \right)}{\left(\frac{0.08}{12} \right)}$$

$$200,000 = \frac{d \left((1.00667)^{360} - 1 \right)}{(0.00667)}$$

$$200,000 = d(1491.57)$$

$$d = \frac{200,000}{1491.57} = \$134.09$$

So you would need to deposit \$134.09 each month to have \$200,000 in 30 years.

Payout Annuities

In the last section you learned about annuities. In an annuity, you start with nothing, put money into an account on a regular basis, and end up with money in your account.

In this section, we will learn about a variation called a **Payout Annuity**. With a payout annuity, you start with money in the account, and pull money out of the account on a regular basis. Any remaining money in the account earns interest. After a fixed amount of time, the account will end up empty.

Payout annuities are typically used after retirement. Perhaps you have saved \$500,000 for retirement, and want to take money out of the account each month to live on. You want the money to last you 20 years. This is a payout annuity. The formula is derived in a similar way as we did for savings annuities. The details are omitted here.

Payout Annuity Formula

$$P_0 = \frac{d \left(1 - \left(1 + \frac{r}{k} \right)^{-Nk} \right)}{\left(\frac{r}{k} \right)}$$

P_0 is the balance in the account at the beginning (starting amount, or principal).
 d is the regular withdrawal (the amount you take out each year, each month, etc.)
 r is the annual interest rate (in decimal form. Example: 5% = 0.05)
 k is the number of compounding periods in one year.
 N is the number of years we plan to take withdrawals

Like with annuities, the compounding frequency is not always explicitly given, but is determined by how often you take the withdrawals.

The most important thing to remember about using this formula is that it assumes that you take money from the account on a regular schedule (every month, year, quarter, etc.) and let the rest sit there earning interest.

Compound interest: One deposit
 Annuity: Many deposits.
 Payout Annuity: Many withdrawals

Example 1. After retiring, you want to be able to take \$1000 every month for a total of 20 years from your retirement account. The account earns 6% interest. How much will you need in your account when you retire?

In this example,
 $d = \$1000$ (the monthly withdrawal)
 $r = 0.06$ (6%)
 $k = 12$ (since we're doing monthly withdrawals, we'll compound monthly)
 $N = 20$, since we were taking withdrawals for 20 years

We're looking for P_0 ; how much money needs to be in the account at the beginning. Putting this into the equation:

$$P_0 = \frac{1000 \left(1 - \left(1 + \frac{0.06}{12} \right)^{-20(12)} \right)}{\left(\frac{0.06}{12} \right)}$$

$$P_0 = \frac{1000 \times \left(1 - (1.005)^{-240} \right)}{(0.005)}$$

$$P_0 = \frac{1000 \times (1 - 0.302)}{(0.005)} = \$139,600$$

So you will need to have \$139,600 in your account when you retire. Notice that you withdrew a total of \$240,000 (\$1000 a month for 240 months). The difference between what you pulled out and what you started with is the interest earned. In this case it is \$240,000 - \$139,600 = \$100,400 in interest.

Example 2. You know you will have \$500,000 in your account when you retire. You want to be able to take monthly withdrawals from the account for a total of 30 years. Your retirement account earns 8% interest. How much will you be able to withdraw each month?

In this example,
 We're looking for d .
 $r = 0.08$ (8%)
 $k = 12$ (since we're withdrawing monthly)
 $N = 30$ (30 years)
 $P_0 = \$500,000$

In this case, we're going to have to set up the equation, and solve for d .

$$500,000 = \frac{d \left(1 - \left(1 + \frac{0.08}{12} \right)^{-30(12)} \right)}{\left(\frac{0.08}{12} \right)}$$

$$500,000 = \frac{d \left(1 - (1.00667)^{-360} \right)}{(0.00667)}$$

$$500,000 = d(136.232)$$

$$d = \frac{500,000}{136.232} = \$3670.21$$

So you would be able to withdraw \$3,670.21 each month for 30 years.

Loans

In the last section, you learned about payout annuities.

In this section, you will learn about conventional loans (also called amortized loans or installment loans). Examples include auto loans and home mortgages. These techniques do not apply to payday loans, add-on loans, or other loan types where the interest is calculated up front.

One great thing about loans is that they use exactly the same formula as a payout annuity. To see why, imagine that you had \$10,000 invested at a bank, and started taking out payments while earning interest as part of a payout annuity, and after 5 years your balance was zero. Flip that around, and imagine that you are acting as the bank, and a car lender is acting as you. The car lender invests \$10,000 in you. Since you're acting as the bank, you pay interest. The car lender takes payments until the balance is zero.

Loans Formula

$$P_0 = \frac{d \left(1 - \left(1 + \frac{r}{k} \right)^{-Nk} \right)}{\left(\frac{r}{k} \right)}$$

In this formula:

P_0 is the balance in the account at the beginning (amount of the loan).

d is your loan payment (your monthly payment, annual payment, etc)

r is the annual interest rate (in decimal form. Example: 5% = 0.05)

k is the number of compounding periods in one year.

N is the length of the loan, in years

Like before, the compounding frequency is not always explicitly given, but is determined by how often you make payments.

The most important thing to remember about using this formula is that it assumes that you make loan payments on a regular schedule (every month, year, quarter, etc.) and are paying interest on the loan.

Compound interest: One deposit

Annuity: Many deposits.

Payout Annuity: Many withdrawals

Loans: Many payments

Example 1. You can afford \$200 per month as a car payment. If you can get an auto loan at 3% interest for 60 months (5 years), how expensive of a car can you afford (in other words, what amount loan can you pay off with \$200 per month)?

In this example,

$d = \$200$ (the monthly payment)

$r = 0.03$ (3%)

$k = 12$ (since we're doing monthly payments, we'll compound monthly)

$N = 5$, since we're making monthly payments for 5 years

We're looking for P_0 , the starting amount of the loan.

$$P_0 = \frac{200 \left(1 - \left(1 + \frac{0.03}{12} \right)^{-5(12)} \right)}{\left(\frac{0.03}{12} \right)}$$

$$P_0 = \frac{200 \left(1 - (1.0025)^{-60} \right)}{(0.0025)}$$

$$P_0 = \frac{200(1 - 0.861)}{(0.0025)} = \$11,120$$

So you can afford a \$11,120 loan. You will pay a total of \$12,000 (\$200 per month for 60 months) to the loan company. The difference between the amount you pay and the amount of the loan is the interest paid. In this case, you're paying \$12,000 - \$11,120 = \$880 interest total.

Example 2. You want to take out a \$140,000 mortgage (home loan). The interest rate on the loan is 6%, and the loan is for 30 years. How much will your monthly payments be?

In this example,

We're looking for d .

$r = 0.06$ (6%)

$k = 12$ (since we're paying monthly)

$N = 30$ (30 years)

$P_0 = \$140,000$ (the loan amount)

In this case, we're going to have to set up the equation, and solve for d .

$$140,000 = \frac{d \left(1 - \left(1 + \frac{0.06}{12} \right)^{-30(12)} \right)}{\left(\frac{0.06}{12} \right)}$$

$$140,000 = \frac{d \left(1 - (1.005)^{-360} \right)}{(0.005)}$$

$$140,000 = d(166.792)$$

$$d = \frac{140,000}{166.792} = \$839.37$$

So you will make payments of \$839.37 per month for 30 years. You're paying a total of \$302,173.20 to the loan company (\$839.37 per month for 360 months). You are paying a total of $\$302,173.20 - \$140,000 = \$162,173.20$ in interest over the life of the loan.

Which equation to use?

When presented with a finance problem (on an exam or in real life), you're usually not told what type of problem it is or which equation to use. Here are some hints on deciding which equation to use based on the wording of the problem.

The easiest types of problem to identify are loans. Loan problems almost always include words like: "loan", "amortize" (the fancy word for loans), "finance (a car)", or "mortgage" (a home loan). Look for these words. If they're there, you're probably looking at a loan problem. To make sure, see if you're given what your monthly (or annual) payment is, or if you're trying to find a monthly payment.

If the problem is not a loan, the next question you want to ask is: "Am I putting money in an account and letting it sit, or am I making regular (monthly/annually/quarterly) payments or withdrawals?" If you're letting the money sit in the account with nothing but interest changing the balance, then you're looking at a compound interest problem. The exception would be bonds and other investments where the interest is not reinvested; in those cases you're looking at simple interest.

If you're making regular payments or withdrawals, the next questions is: "Am I putting money into the account, or am I pulling money out?" If you're putting money into the account on a regular basis (monthly/annually/quarterly) then you're looking at a basic Annuity problem. Basic annuities are when you are saving money. Usually in an annuity problem, your account starts empty, and has money in the future. If you're pulling money out of the account on a regular basis, then you're looking at a Payout Annuity problem. Payout annuities are used for things like retirement income, where you start with money in your account, pull money out on a regular basis, and your account ends up empty in the future.

Remember, the most important part of answering any kind of question, money or otherwise, is first to correctly identify what the question is really asking, and to determine what approach will best allow you to solve the problem.

Using your Calculator

When we need to calculate something like 5^3 it is easy enough to just multiply $5 \cdot 5 \cdot 5 = 125$. But when we need to calculate something like 1.005^{360} , it would be very tedious to calculate this by multiplying 1.005 by itself 360 times! So to make things easier, we can harness the power of our scientific calculators.

Most scientific calculators have a button for exponents. It is typically either labeled like:

$$\boxed{\wedge}, \boxed{y^x}, \text{ or } \boxed{x^y}.$$

To evaluate 1.005^{360} we'd type $1.005 \boxed{\wedge} 360$, or $1.005 \boxed{y^x} 360$. Try it out - you should get something around 6.022575.

With some problems, you need to raise numbers to negative powers. Most calculators have a separate button for negating a number that is different than the subtraction button. Some calculators label this $\boxed{(-)}$, some with $\boxed{+/-}$. The button is often near the = key or the decimal point.

If your calculator displays operations on it (typically a calculator with multiline display), to calculate 1.03^{-10} you'd type something like: $1.03 \boxed{\wedge} \boxed{(-)} 10$

If your calculator only shows one value at a time, then usually you hit the (-) key after a number to negate it, so you'd hit: $1.03 \boxed{y^x} 10 \boxed{(-)} =$

Give it a try - you should get $1.03^{-10} = 0.744094$

A note on rounding

It is important to be very careful about rounding when calculating things with exponents. In general, you want to keep as many decimals during calculations as you can. Be sure to keep at least 3 significant digits (numbers after any leading zeros). So rounding 0.00012345 to 0.000123 will usually give you a "close enough" answer, but keeping more digits is always better.

To see why, suppose you were investing \$1000 at 5% interest compounded monthly for 30 years.

$P_0 = \$1000$ (the initial deposit)

$r = 0.05$ (5%)

$k = 12$ (12 months in 1 year)

$N = 30$, since we're looking for P_{30}

If we first compute r/k , we find $0.05/12 = 0.00416666666667$

Here is the effect of rounding this to different values:

<i>r/k</i> rounded to:	Gives P_{30} to be:	Error
0.004	\$4208.59	\$259.15
0.0042	\$4521.45	\$53.71
0.00417	\$4473.09	\$5.35
0.004167	\$4468.28	\$0.54
0.0041667	\$4467.80	\$0.06
no rounding	\$4467.74	

If you're working in a bank, of course you wouldn't round at all. But for our purposes, the answer we got by rounding to three significant digits is close enough - \$5 off of \$4500 isn't too bad. Certainly keeping that fourth decimal place wouldn't have hurt.

Exercises

Skills

1. A friend lends you \$200 for a week, which you agree to repay with 5% interest. How much will you have to repay?
2. Suppose you obtain a \$3,000 T-note with a 3% annual rate, paid quarterly, with maturity in 5 years. How much interest will you earn?
3. A T-bill is a type of bond that is sold at a discount over the face value. For example, suppose you buy a 13-week T-bill with a face value of \$10,000 for \$9,800. This means that in 13 weeks, the government will give you the face value, earning you \$200. What annual interest rate have you earned?
4. Suppose you are looking to buy a \$5000 face value 26-week T-bill. If you want to earn at least 1% interest, what is the most you should pay for the T-bill?
5. You deposit \$300 in an account earning 5% interest compounded annually. How much will you have in the account in 10 years?
6. How much will \$1000 deposited in an account earning 7% interest compounded annually be worth in 20 years?
7. You deposit \$2000 in an account earning 3% interest compounded monthly.
 - a. How much will you have in the account in 20 years?
 - b. How much interest will you earn?
8. You deposit \$10,000 in an account earning 4% interest compounded monthly.
 - a. How much will you have in the account in 25 years?
 - b. How much interest will you earn?
9. How much would you need to deposit in an account now in order to have \$6,000 in the account in 8 years? Assume the account earns 6% interest compounded monthly.
10. How much would you need to deposit in an account now in order to have \$20,000 in the account in 4 years? Assume the account earns 5% interest.
11. You deposit \$200 each month into an account earning 3% interest compounded monthly.
 - a. How much will you have in the account in 30 years?
 - b. How much total money will you put into the account?
 - c. How much total interest will you earn?
12. You deposit \$1000 each year into an account earning 8% compounded annually.
 - a. How much will you have in the account in 10 years?
 - b. How much total money will you put into the account?
 - c. How much total interest will you earn?

13. Jose has determined he needs to have \$800,000 for retirement in 30 years. His account earns 6% interest.
 - a. How much would you need to deposit in the account each month?
 - b. How much total money will you put into the account?
 - c. How much total interest will you earn?
14. You wish to have \$3000 in 2 years to buy a fancy new stereo system. How much should you deposit each quarter into an account paying 8% compounded quarterly?
15. You want to be able to withdraw \$30,000 each year for 25 years. Your account earns 8% interest.
 - a. How much do you need in your account at the beginning
 - b. How much total money will you pull out of the account?
 - c. How much of that money is interest?
16. How much money will I need to have at retirement so I can withdraw \$60,000 a year for 20 years from an account earning 8% compounded annually?
 - a. How much do you need in your account at the beginning
 - b. How much total money will you pull out of the account?
 - c. How much of that money is interest?
17. You have \$500,000 saved for retirement. Your account earns 6% interest. How much will you be able to pull out each month, if you want to be able to take withdrawals for 20 years?
18. Loren already knows that he will have \$500,000 when he retires. If he sets up a payout annuity for 30 years in an account paying 10% interest, how much could the annuity provide each month?
19. You can afford a \$700 per month mortgage payment. You've found a 30 year loan at 5% interest.
 - a. How big of a loan can you afford?
 - b. How much total money will you pay the loan company?
 - c. How much of that money is interest?
20. Marie can afford a \$250 per month car payment. She's found a 5 year loan at 7% interest.
 - a. How expensive of a car can she afford?
 - b. How much total money will she pay the loan company?
 - c. How much of that money is interest?
21. You want to buy a \$25,000 car. The company is offering a 2% interest rate for 48 months (4 years). What will your monthly payments be?
22. You decide finance a \$12,000 car at 3% compounded monthly for 4 years. What will your monthly payments be? How much interest will you pay over the life of the loan?

23. You want to buy a \$200,000 home. You plan to pay 10% as a down payment, and take out a 30 year loan for the rest.
- How much is the loan amount going to be?
 - What will your monthly payments be if the interest rate is 5%?
 - What will your monthly payments be if the interest rate is 6%?
24. Lynn bought a \$300,000 house, paying 10% down, and financing the rest at 6% interest for 30 years.
- Find her monthly payments.
 - How much interest will she pay over the life of the loan?

Concepts

25. Suppose you invest \$50 a month for 5 years into an account earning 8% compounded monthly. After 5 years, you leave the money, without making additional deposits, in the account for another 25 years. How much will you have in the end?
26. Suppose you put off making investments for the first 5 years, and instead made deposits of \$50 a month for 25 years into an account earning 8% compounded monthly. How much will you have in the end?
27. Mike plans to make contributions to his retirement account for 15 years. After the last contribution, he will start withdrawing \$10,000 a quarter for 10 years. Assuming Mike's account earns 8% compounded quarterly, how large must his quarterly contributions be during the first 15 years, in order to accomplish his goal?
28. Kendra wants to be able to make withdrawals of \$60,000 a year for 30 years after retiring in 35 years. How much will she have to save each year up until retirement if her account earns 7% interest?

Exploration

29. Pay day loans are short term loans that you take out against future paychecks: The company advances you money against a future paycheck. Either visit a pay day loan company, or look one up online. Be forewarned that many companies do not make their fees obvious, so you might need to do some digging or look at several companies.
- Explain the general method by which the loan works.
 - We will assume that we need to borrow \$500 and that we will pay back the loan in 14 days. Determine the total amount that you would need to pay back and the effective loan rate. The effective loan rate is the percentage of the original loan amount that you pay back. It is not the same as the APR (annual rate) that is probably published.
 - If you cannot pay back the loan after 14 days, you will need to get an extension for another 14 days. Determine the fees for an extension, determine the total amount you will be paying for the now 28 day loan, and compute the effective loan rate.

30. Suppose that 10 years ago you bought a home for \$110,000, paying 10% as a down payment, and financing the rest at 9% interest for 30 years.
- a. Let's consider your existing mortgage:
 - i. How much money did you pay as your down payment?
 - ii. How much money was your mortgage (loan) for?
 - iii. What is your current monthly payment?
 - iv. How much total interest will you pay over the life of the loan?
 - b. This year, you check your loan balance. Only part of your payments have been going to pay down the loan; the rest has been going towards interest. You see that you still have \$88,536 left to pay on your loan. Your house is now valued at \$150,000.
 - i. How much of the loan have you paid off? (i.e., how much have you reduced the loan balance by? Keep in mind that interest is charged each month - it's not part of the loan balance.)
 - ii. How much money have you paid to the loan company so far?
 - iii. How much interest have you paid so far?
 - iv. How much equity do you have in your home (equity is value minus remaining debt)
 - c. Since interest rates have dropped, you consider refinancing your mortgage at a lower 6% rate.
 - i. If you took out a new 30 year mortgage at 6% for your remaining loan balance, what would your new monthly payments be?
 - ii. How much interest will you pay over the life of the new loan?
 - d. Notice that if you refinance, you are going to be making payments on your home for another 30 years. In addition to the 10 years you've already been paying, that's 40 years total.
 - i. How much will you save each month because of the lower monthly payment?
 - ii. How much total interest will you be paying (you need to consider the amount from 2c and 3b)
 - iii. Does it make sense to refinance? (there isn't a correct answer to this question. Just give your opinion and your reason)