Chapter 7: Sets

It is natural for us to classify items into groups, or sets, and consider how those sets overlap with each other. We can use these sets understand relationships between groups, and to analyze survey data.

Section 7.1: Sets

Basics

An art collector might own a collection of paintings, while a music lover might keep a collection of CDs. Any collection of items can form a **set**.

Set

A **set** is a collection of distinct objects, called **elements** of the set

A set can be defined by describing the contents, or by listing the elements of the set, enclosed in curly brackets.

Example 1

Some examples of sets defined by describing the contents:
a) The set of all even numbers
b) The set of all books written about travel to Chile

Some examples of sets defined by listing the elements of the set:
a) {1, 3, 9, 12}
b) {red, orange, yellow, green, blue, indigo, purple}

A set simply specifies the contents; order is not important. The set represented by {1, 2, 3} is equivalent to the set {3, 1, 2}.

Notation

Commonly, we will use a variable to represent a set, to make it easier to refer to that set later.

The symbol ∊ means "is an element of".

A set that contains no elements, { }, is called the **empty set** and is notated ∅

Example 2

Let $A = \{1, 2, 3, 4\}$

To notate that 2 is element of the set, we'd write $2 \in A$

Sometimes a collection might not contain all the elements of a set. For example, Chris owns three Madonna albums. While Chris's collection is a set, we can also say it is a **subset** of the larger set of all Madonna albums.

Subset

A **subset** of a set A is another set that contains only elements from the set A, but may not contain all the elements of A.

If B is a subset of A, we write $B \subseteq A$

A **proper subset** is a subset that is not identical to the original set – it contains fewer elements.

If B is a proper subset of A, we write $B \subset A$

Example 3

Consider these three sets
A = the set of all even numbers $B = \{2, 4, 6\}$ $C = \{2, 3, 4, 6\}$

Here $B \subset A$ since every element of B is also an even number, so is an element of A.

More formally, we could say $B \subset A$ since if $x \in B$, then $x \in A$.

It is also true that $B \subset C$.

C is not a subset of A, since C contains an element, 3, that is not contained in A

Example 4

Suppose a set contains the plays "Much Ado About Nothing", "MacBeth", and "A Midsummer's Night Dream". What is a larger set this might be a subset of?

There are many possible answers here. One would be the set of plays by Shakespeare. This is also a subset of the set of all plays ever written. It is also a subset of all British literature.

Try it Now

1. The set $A = \{1, 3, 5\}$. What is a larger set this might be a subset of?

Union, Intersection, and Complement

Commonly sets interact. For example, you and a new roommate decide to have a house party, and you both invite your circle of friends. At this party, two sets are being combined, though it might turn out that there are some friends that were in both sets.

Union, Intersection, and Complement

The **union** of two sets contains all the elements contained in either set (or both sets).
The union is notated $A \cup B$.
More formally, $x \in A \cup B$ if $x \in A$ or $x \in B$ (or both)

The **intersection** of two sets contains only the elements that are in both sets.
The intersection is notated $A \cap B$.
More formally, $x \in A \cap B$ if $x \in A$ and $x \in B$

The **complement** of a set *A* contains everything that is *not* in the set *A*.
The complement is notated A', or A^c, or sometimes $\sim A$.

Example 5

Consider the sets: A = {red, green, blue} B = {red, yellow, orange}
C = {red, orange, yellow, green, blue, purple}

a) Find $A \cup B$

The union contains all the elements in either set: $A \cup B$ = {red, green, blue, yellow, orange}
Notice we only list red once.

b) Find $A \cap B$

The intersection contains all the elements in both sets: $A \cap B$ = {red}

c) Find $A^c \cap C$

Here we're looking for all the elements that are *not* in set *A* and are also in *C*.
$A^c \cap C$ = {orange, yellow, purple}

Try it Now

2. Using the sets from the previous example, find $A \cup C$ and $B^c \cap A$

Notice that in the example above, it would be hard to just ask for A^c, since everything from the color fuchsia to puppies and peanut butter are included in the complement of the set. For this reason, complements are usually only used with intersections, or when we have a universal set in place.

Universal Set

A **universal set** is a set that contains all the elements we are interested in. This would have to be defined by the context.

A complement is relative to the universal set, so A^c contains all the elements in the universal set that are not in A.

Example 6

a) If we were discussing searching for books, the universal set might be all the books in the library.
b) If we were grouping your Facebook friends, the universal set would be all your Facebook friends.
c) If you were working with sets of numbers, the universal set might be all whole numbers, all integers, or all real numbers

Example 7

Suppose the universal set is U = all whole numbers from 1 to 9. If $A = \{1, 2, 4\}$, then

$A^c = \{3, 5, 6, 7, 8, 9\}$.

As we saw earlier with the expression $A^c \cap C$, set operations can be grouped together. Grouping symbols can be used like they are with arithmetic – to force an order of operations.

Example 8

Suppose H = {cat, dog, rabbit, mouse}, F = {dog, cow, duck, pig, rabbit}
W = {duck, rabbit, deer, frog, mouse}

a) Find $(H \cap F) \cup W$

We start with the intersection: $H \cap F$ = {dog, rabbit}
Now we union that result with W: $(H \cap F) \cup W$ = {dog, duck, rabbit, deer, frog, mouse}

b) Find $H \cap (F \cup W)$

We start with the union: $F \cup W$ = {dog, cow, rabbit, duck, pig, deer, frog, mouse}
Now we intersect that result with H: $H \cap (F \cup W)$ = {dog, rabbit, mouse}

c) Find $(H \cap F)^c \cap W$

We start with the intersection: $H \cap F$ = {dog, rabbit}
Now we want to find the elements of W that are *not* in $H \cap F$
$(H \cap F)^c \cap W$ = {duck, deer, frog, mouse}

Important Topics of this Section

Sets, elements of sets, subsets
Union, intersection, and complement of sets
Universal set, empty set

Try it Now Answers

1. There are several answers: The set of all odd numbers less than 10. The set of all odd numbers. The set of all integers. The set of all real numbers.

2. $A \cup C$ = {red, orange, yellow, green, blue purple}
$B^c \cap A$ = {green, blue}