Chapter 4 Exercises

1. $F(x,y) = x^2 - y^2$. Find

a) $F(0,4)$ b) $F(4,0)$ c) $F(x,4)$ d) $F(4,y)$

e) $F(800,800)$ f) $F(x,x)$ g) $F(x,-x)$

2. $g(s,t) = \sqrt{st^2}$. Find

a) $g(1,9)$ b) $g(9,1)$ c) $g(1,t)$ d) $g(s,9)$ e) $g(w,z+1)$

3. Let $f(x,y,z,w) = x^2 - \frac{1}{zw} + xyz^2$. Evaluate $f(1,2,3,4)$.

4. Let $f(x,y,z,w) = \sqrt{xy} - w^2 + 102yz$. Evaluate $f(1,2,3,4)$.

5. Here is a table showing the function $A(t,r)$

t ↓ r →	.03	.04	.05	.06	.07
1	30.45	40.81	51.27	61.84	72.51
2	61.84	83.29	105.17	127.50	150.27
3	94.17	127.50	161.83	197.22	233.68

a) Find $A(2,.05)$

b) Find $A(.05,.2)$

c) Is $A(t,.06)$ an increasing or decreasing function of t ?

d) Is $A(3,r)$ an increasing or decreasing function of r?

6. Here is a table showing values for the function $H(t,h)$.

t ↓ h →	100	150	200
0	100	150	200
1	110.1	160.1	210.1
2	110.4	160.4	210.4
3	100.9	150.9	200.9
4	81.6	131.6	181.6
5	52.5	102.5	152.5

a) Is $H(t,150)$ an increasing or decreasing function of t ?

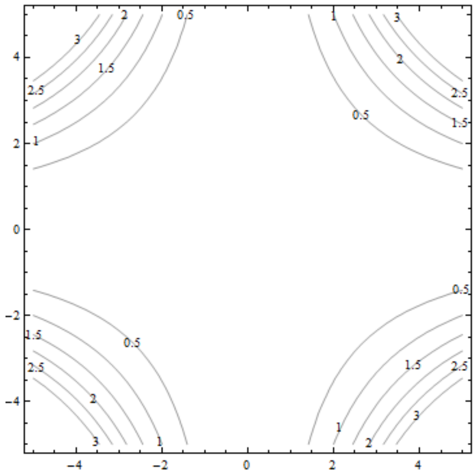
b) Is $H(4,h)$ an increasing or decreasing function of h?

c) Fill in the blanks: The maximum value shown on this table is $H(\_\_\_,\_\_\_) = \_\_\_\_$.

d) Fill in the blanks: The minimum value shown on this table is $H(\_\_\_,\_\_\_) = \_\_\_\_$.

For problems 7 through 12. Match the contour diagram to the computer-generated, perspective drawing (a through f) it matches. Briefly explain your answer.

7.



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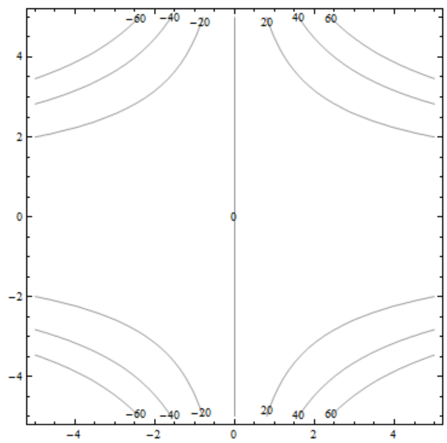

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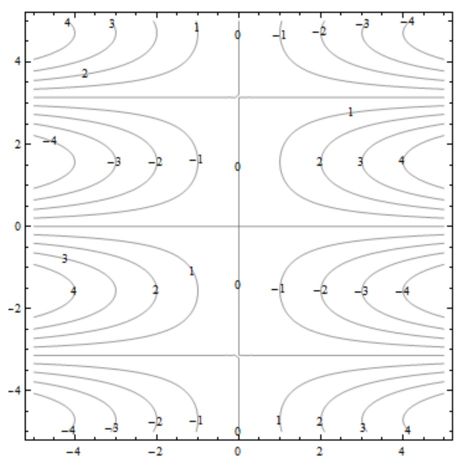

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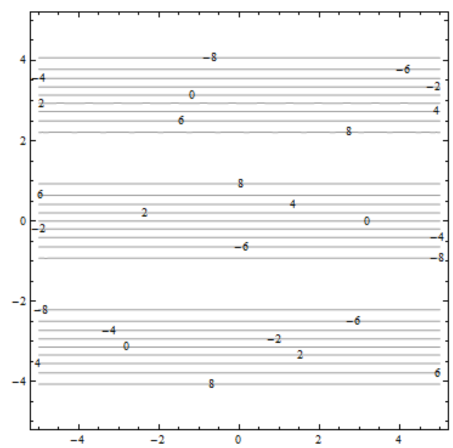

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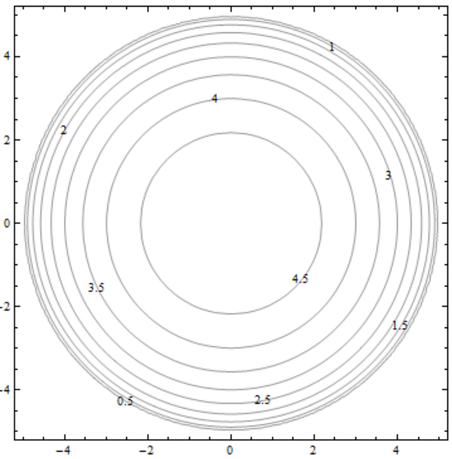

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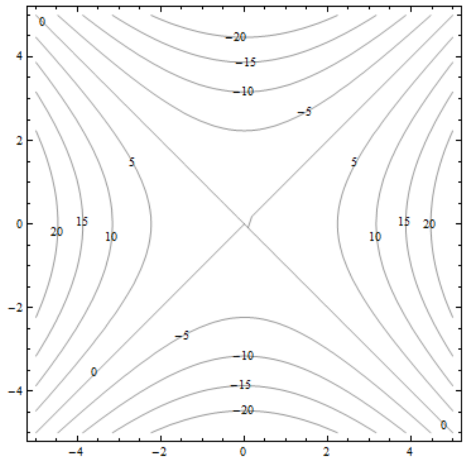
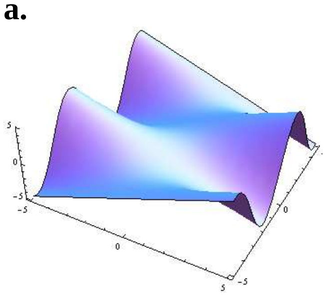
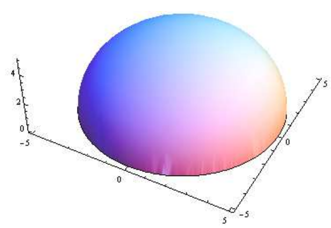


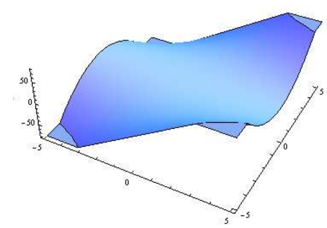
a.



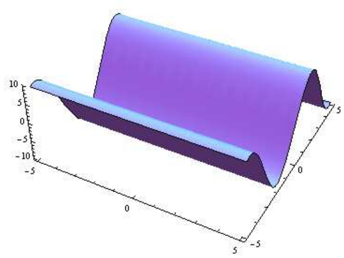
b.



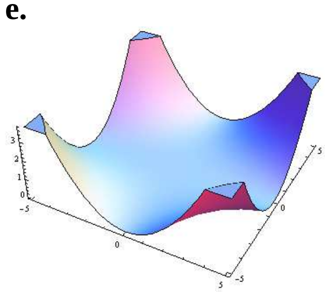
c.



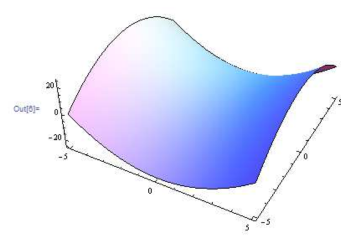
d.



e.

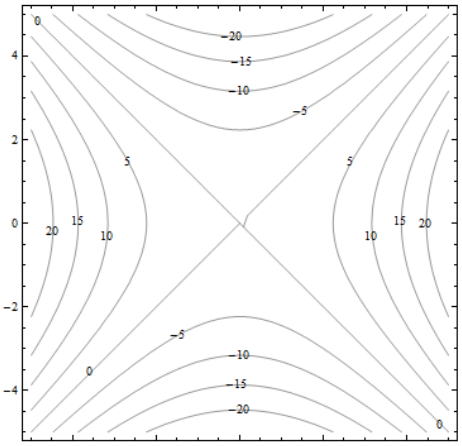


f.



For problems 13 through 18. Match the contour diagram to the equation (a through f) it matches. Briefly explain your answer.

13.



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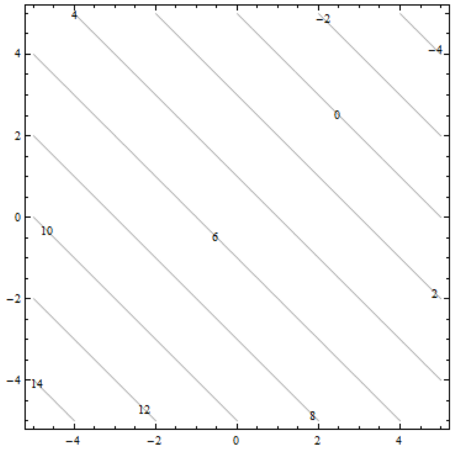

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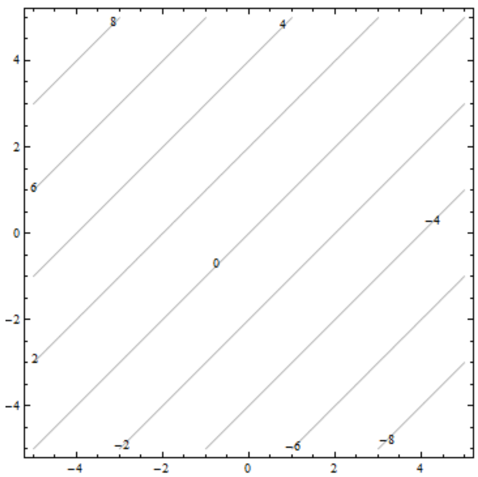

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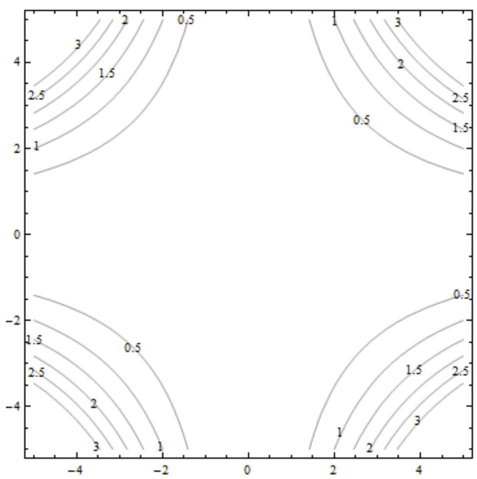

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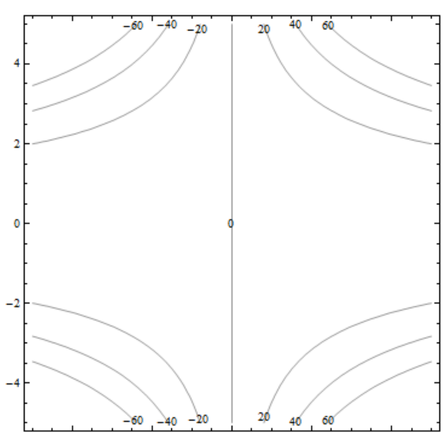

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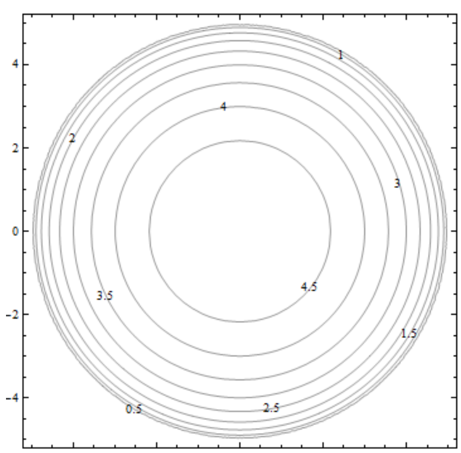

a. $f(x, y) = y - x$ b. $f(x, y) = xy^2$

c. $f(x, y) = \sqrt{25 - x^2 - y^2}$ d. $f(x, y) = 5 - x - y$

e. $f(x, y) = 0.01x^2y^2$ f. $f(x, y) = x^2 - y^2$

19. The contour diagram shown is for a function M(x, y). Use the diagram to answer the following:

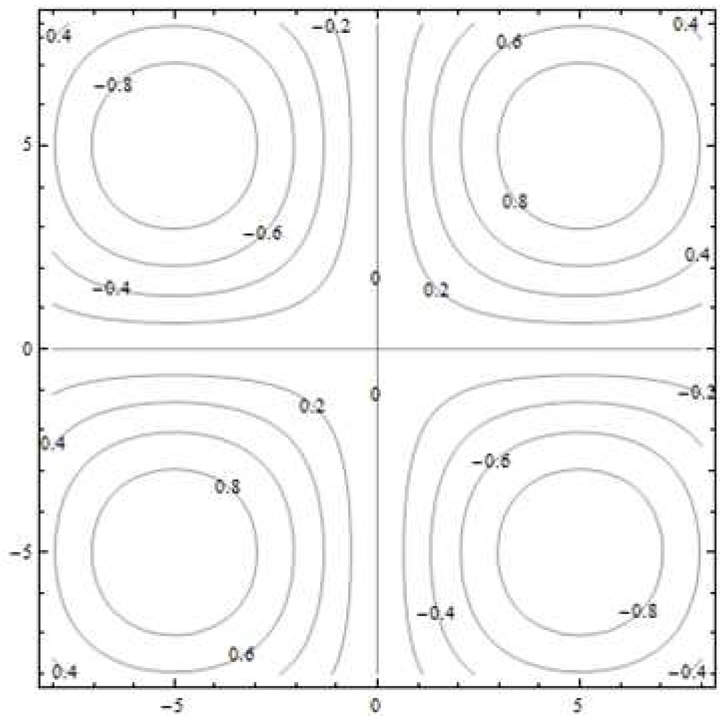

Figure 1

a) Estimate M(1, 3)
b) Estimate M(3, 1)
c) Is M(x, 3) an increasing or decreasing function of x?
d) Is M(3, y) an increasing or decreasing function of y?
e) Find a value of c so that M(c, y) is a constant function of y.

20. The contour diagram shown is for a function G(x, y). Use the diagram to answer the following:

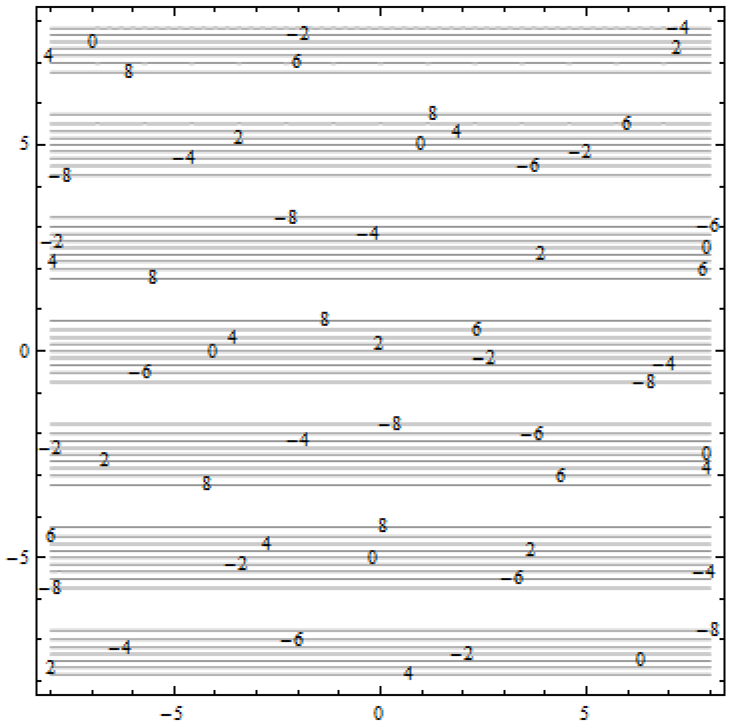

Figure 2

a) Estimate G(2, 3)
b) Suppose you travel north (in the direction of increasing y) along the surface, starting above (2, 3). Describe your journey.
c) Suppose you travel east (in the direction of increasing x) along the surface, starting above (2, 3). Describe your journey.

For problems 21 through 35, find f_x and f_y for the function given

21. $f(x,y)=x^2-5y^2$

22. $f(x,y)=\dfrac{x^2-5y^2}{x+4}$

23. $f(x,y)=e^{x+6y}$

24. $f(x,y)=(x^2-5y^2)e^x$

25. $f(x,y)=(x^2-5y^2)\left(\dfrac{1}{3y}+4\right)$

26. $f(x,y)=x$

27. $f(x,y)=6$

28. $f(x,y)=\ln(xy+2x-6y)$

29. $f(x,y)=\dfrac{x^2-5y^2}{y^4-5x^4}$

30. $f(x,y)=e^{\sqrt{x-4y}}(x-4y)$

31. $f(x,y)=y^5e^x$

32. $f(x,y)=\dfrac{1}{16xy}$

33. $f(x,y)=(x+e^y)^7$

34. $f(x,y)=x^4+4x^3y-6x^2y^2-4xy^3+y^4$

35. $f(x,y)=\sqrt{x+\sqrt{y}}$

For problems 36 through 46, find f_{xx}, f_{yy}, f_{xy} and f_{yx} for the function given. Confirm that $f_{xy}=f_{yx}$.

36. $f(x,y)=x^2-5y^2$

37. $f(x,y)=x^4+4x^3y-6x^2y^2-4xy^3+y^4$

38. $f(x,y)=5x^2y^2$

39. $f(x,y)=e^{x+6y}$

40. $f(x,y)=\ln(xy+2x-6y)$

41. $f(x,y)=\dfrac{x^2}{y^4-5}$

42. Given the function $g(x,y)=\left(x^4-e^x\right)\left(x-\dfrac{1}{x}\right)y^2$, find g_{xxyyy}. Hint – think about which order to find these partial derivatives in; is there a way to save yourself some work?

43. Here is a table showing the function $A(t,r)$

t ↓ r →	.03	.04	.05	.06	.07
1	30.45	40.81	51.27	61.84	72.51
2	61.84	83.29	105.17	127.50	150.27
3	94.17	127.50	161.83	197.22	233.68

a. Estimate $A_t(2,.05)$.
b. Estimate $A_r(2,.05)$
c. Use your answers to parts a and b to estimate the value of $A(2.5,.054)$
d. The values in the table came from $A(t,r)=1000\left(e^{rt}-1\right)$, which shows the interest earned if 1000 dollars is deposited in an account earning *r* annual interest, compounded continuously, and left there for *t* years. How close are your estimates from parts a, b, and c?

44. Here is a table showing values for the function $H(t,h)$.

t ↓ h →	100	150	200
0	100	150	200
1	110.1	160.1	210.1
2	110.4	160.4	210.4
3	100.9	150.9	200.9
4	81.6	131.6	181.6
5	52.5	102.5	152.5

a. Estimate the value of $\frac{\partial H}{dt}$ at (3, 150).

b. Estimate the value of $\frac{\partial H}{dh}$ at (3, 150).

c. Use your answers to parts a and b to estimate the value of $H(2.6{,}156)$.

d. The values in the table came from $H(t,h) = h + 15t - 4.9t^2$, which gives the height in meters above the ground after *t* seconds of an object that is thrown upward from an initial height of *h* meters with an initial velocity of 15 meters per second. How close are your estimates from parts a, b, and c?

45. Find the critical points of $f(x,y) = y^3 - x^3 + 15x^2 - 12y + 12$ and use the Second Derivative Test to classify them. If the test fails, say "the test fails."

46. Find the critical points of $f(x,y) = 2xy - x^2 - 2y^2 + 6x + 4$ and use the Second Derivative Test to classify them. If the test fails, say "the test fails."

47. Find the critical points of $f(x,y) = y^2 - 4\ln(x) + 4x$ and use the Second Derivative Test to classify them. If the test fails, say "the test fails."

48. Find the critical points of $f(x,y) = xy - 6x^2 + 3x - y + 2$ and use the Second Derivative Test to classify them. If the test fails, say "the test fails."

49. The origin is a critical point for the function $f(x,y) = x^3 + y^3$, and D = 0 there. That is, the Second Derivative Test fails. Use what you know about shapes of functions to decide if there is a local minimum, local maximum, or saddle point for this function at (0, 0).

50. The origin is a critical point for the function $f(x,y) = 15 - x^2y^2$, and D = 0 there. That is, the Second Derivative Test fails. Use what you know about shapes of functions to decide if there is a local minimum, local maximum, or saddle point for this function at (0, 0).

For problems 51 through 56, find all local maxima, minima, and saddle points for the function.

51. $f(x,y) = xy - 5x^2 - 5y^2 + 33y$

52. $f(x,y)=10xy-x^2-y^2+3x$

53. $f(x,y)=x^3+y^3-3xy$

54. $f(x,y)=5x^2-4xy+2y^2+4x-4y+10$

55. $f(x,y)=y^2e^x+x^2$

56. $f(x,y)=xy+2x-\ln(x^2y)$, for $x>0$ and $y>0$.

57. The demand functions for two products are given below. p_1, p_2, q_1,and q_2 are the prices (in dollars) and quantities for products 1 and 2.

$$q_1 = 200-3p_1+p_2$$
$$q_2 = 150+p_1-2p_2$$

a. Are these two products complementary goods or substitute goods?
b. What is the quantity demanded for each when the price for product 1 is $20 per item and the price for product 2 is $30 per item?
c. Write a function $R(p_1,p_2)$ that expresses the total revenue from these two products.
d. Find the price and quantity for each product that maximizes the total revenue.

58. Suppose the demand functions for two products are $q_1 = f(p_1, p_2)$ and $q_2 = g(p_1, p_2)$, where p_1, p_2, q_1, and q_2 are the prices (in dollars) and quantities for products 1 and 2. Consider the four partial derivatives $\frac{\partial q_1}{\partial p_1}, \frac{\partial q_1}{\partial p_2}, \frac{\partial q_2}{\partial p_1}$, and $\frac{\partial q_2}{\partial p_2}$. Tell the sign of each of these partial derivatives if

a. the products are complementary goods.
b. the products are substitute goods.