

## Section 4: Exponents

The Laws of Exponents let you rewrite algebraic expressions that involve exponents. The last three listed here are really definitions rather than rules.

### Laws of Exponents:

All variables here represent real numbers and all variables in denominators are nonzero.

$$1) x^a \cdot x^b = x^{a+b}$$

$$2) \frac{x^a}{x^b} = x^{a-b}$$

$$3) (x^a)^b = x^{ab}$$

$$4) (xy)^a = x^a y^a$$

$$5) \left(\frac{x}{y}\right)^b = \frac{x^b}{y^b}$$

$$6) x^0 = 1, \text{ provided } x \neq 0$$

$$7) x^{-n} = \frac{1}{x^n}, \text{ provided } x \neq 0$$

$$8) x^{1/n} = \sqrt[n]{x}, \text{ provided } x \neq 0$$

### Example 1

Simplify  $(2x^2)^3(4x)$ .

We'll begin by simplifying the  $(2x^2)^3$  portion. Using property #4, we can write

$$2^3(x^2)^3(4x)$$

Evaluate  $2^3$ , and use property #3

$$8x^6(4x)$$

Multiply the constants, and use property #1, recalling  $x = x^1$

$$32x^7$$

Being able to work with negative and fractional exponents will be very important later in this course.

### Example 2

Rewrite  $\frac{5}{x^3}$  using negative exponents.

Since  $x^{-n} = \frac{1}{x^n}$ , then  $x^{-3} = \frac{1}{x^3}$ .

$$\frac{5}{x^3} = 5x^{-3}$$

**Example 3**

Simplify as much as possible and write your answer using only positive exponents:  $\left(\frac{x^{-2}}{y^{-3}}\right)^2$

$$\left(\frac{x^{-2}}{y^{-3}}\right)^2 = \frac{(x^{-2})^2}{(y^{-3})^2} = \frac{x^{-4}}{y^{-6}} = \frac{y^6}{x^4}$$

**Example 4**

Rewrite  $4\sqrt{x} - \frac{3}{\sqrt{x}}$  using exponents.

A square root is a radical with index of two. In other words,  $\sqrt{x} = \sqrt[2]{x}$ . Using the exponent rule give above,  $\sqrt{x} = \sqrt[2]{x} = x^{1/2}$ . Rewriting the square roots using the fractional exponent,

$$4\sqrt{x} - \frac{3}{\sqrt{x}} = 4x^{1/2} - \frac{3}{x^{1/2}}$$

Now we can use the negative exponent rule to rewrite the second term in the expression.

$$4x^{1/2} - \frac{3}{x^{1/2}} = 4x^{1/2} - 3x^{-1/2}$$

**Example 5**

Rewrite using only positive exponents:  $(\sqrt{p^5})^{-1/3}$

$$(\sqrt{p^5})^{-1/3} = ((p^5)^{1/2})^{-1/3} = p^{-5/6} = \frac{1}{p^{5/6}}$$

**Example 6**

Rewrite  $x^{-4/3}$  as a radical.

First, we can use the negative exponent rule to rewrite this as  $\frac{1}{x^{4/3}}$ .

Since  $\frac{4}{3} = 4 \cdot \frac{1}{3}$ , we can use laws of exponents to rewrite this as  $\frac{1}{(x^{1/3})^4}$ .

Now using the radical equivalence, we can rewrite this as  $\frac{1}{(\sqrt[3]{x})^4}$ .

## 1.4 Exercises

Simplify each expression

1.  $x^3x^5$

2.  $x^4x^2$

3.  $(x^3)^4$

4.  $(x^7)^2$

5.  $(2x^2)^3 x^4$

6.  $(5x^4)^2 x^5$

7.  $\frac{(3x^2)^2}{6x^3}$

8.  $\frac{5x(4x)^2}{2x^2}$

Simplify, and rewrite without negative exponents

9.  $4x^{-3}$

10.  $2x^{-5}$

11.  $x^{-4}x^2$

12.  $x^{-2}x$

13.  $\frac{5x^{-3}}{2x^{-6}}$

14.  $\frac{2x^{-4}}{6x^{-2}}$

Rewrite using negative or fractional exponents

15.  $\frac{4}{x^{-5}}$

16.  $\frac{4}{x^{-3}}$

17.  $3\sqrt{x}$

18.  $\sqrt[4]{x}$

19.  $\frac{4}{\sqrt[3]{x}}$

20.  $\frac{1}{5\sqrt{x}}$

Rewrite as a radical

21.  $4x^{-1/2}$

22.  $5x^{-1/3}$

23.  $2x^{1/3}$

24.  $5x^{3/2}$