Introduction

A Preview of Calculus

Calculus was first developed more than three hundred years ago by Sir Isaac Newton and Gottfried Leibniz to help them describe and understand the rules governing the motion of planets and moons. Since then, thousands of other men and women have refined the basic ideas of calculus, developed new techniques to make the calculations easier, and found ways to apply calculus to problems besides planetary motion. Perhaps most importantly, they have used calculus to help understand a wide variety of physical, biological, economic and social phenomena and to describe and solve problems in those areas.

Part of the beauty of calculus is that it is based on a few very simple ideas. Part of the power of calculus is that these simple ideas can help us understand, describe, and solve problems in a variety of fields.

About this book

Chapter 0 Introduction and Preliminaries gives a brief introduction to calculus in general and this course in particular.

Chapter 1 Review contains review material that you should recall before we begin calculus.

Chapter 2 The Derivative builds on the precalculus idea of the slope of a line to let us find and use rates of change in many situations.

Chapter 3 The Integral builds on the precalculus idea of the area of a rectangle to let us find accumulated change in more complicated and interesting settings.

Chapter 4 Functions of Two Variables extends the calculus ideas of chapter 2 to functions of more than one variable.

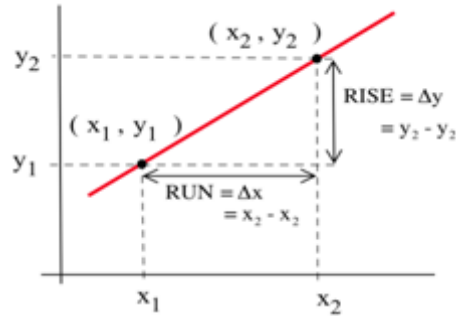## Two Powerful Precalculus Ideas

Calculus is a way to extend some powerful precalculus ideas to more complicated functions. In this course, we will extend two powerful ideas – slope and area.
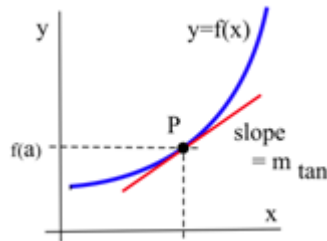
Slope

The slope of a line measures how fast a line rises or falls as we move from left to right along the line. It measures the rate of change of the y-coordinate with respect to changes in the x-coordinate. If the line represents the distance traveled over time, for example, then its slope

represents the velocity. In Figure 1, you can remind yourself of how we calculate slope using two points on the line:

$$m = \text{slope from P to Q} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$
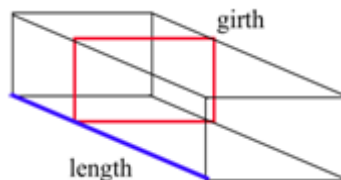

We would like to be able to get that same sort of information (how fast the curve rises or falls, velocity from distance) even if the graph is not a straight line. But what happens if we try to find the slope of a curve, as in Figure 1? We need two points in order to determine the slope of a line. How can we find a slope of a curve, at just one point?

In Chapter 2, we'll extend the idea of slope to curves, and we'll be able to use it to answer many different kinds of questions.

Here is an example of a question that can be answered with slope (in Chapter 2):

Ex: The US Post Office requires that the length plus the girth (Figure 3) of a package not exceed 84 inches. What are the dimensions of the rectangular box with the largest volume that the Post Office will accept?

Area

It's easy to compute the area of a rectangle, or something made of rectangles. But what if the object we want to find the area of is irregular and curvy? We don't have formulas (like length x width) for every shape, and we don't want to. We want a way to find the area of any shape.

It turns out that area tells us much more than simply how much carpet we will need to buy to cover a region. And in Chapter 3, we'll extend the idea of area and use it to answer many different kinds of questions.

Here is an example of a question that can be answered with area (in Chapter 3):

Ex: For a certain product, the demand function is given by $p = 300 - 4q^2$ and the supply function is given by $p = 3 + q^2$, where p is the price in dollars for the product and q is the quantity. At the equilibrium point, find the total gains from trade (consumer surplus + producer surplus).

How is Business Calculus Different?

Students who plan to go into science, engineering, or mathematics take a year-long sequence of classes that cover many of the same topics as we do in our one-quarter or one-semester course. Here are some of the differences:

No trigonometry

We will not be using trigonometry at all in this course. The scientists and engineers need trigonometry frequently, and so a great deal of the engineering calculus course is devoted to trigonometric functions and the situations they can model.

The applications are different

The scientists and engineers learn how to apply calculus to physics problems, such as work. They do a lot of geometric applications, like finding minimum distances, volumes of revolution, or arclengths. In this class, we will do only a few of these (distance/velocity problems, areas between curves). On the other hand, we will learn to apply calculus in some economic and business settings, like maximizing profit or minimizing average cost, finding elasticity of demand, or finding the present value of a continuous income stream. These are applications that are seldom seen in a course for engineers.

Fewer theorems, no proofs

The focus of this course is applications rather than theory. In this course, we will use the results of some theorems, but we won't prove any of them. When you finish this course, you should be able to solve many kinds of problems using calculus, but you won't be prepared to go on to higher mathematics.

Less algebra

In this class, you will not need clever algebra. If you need to solve an equation, it will either be relatively simple, or you can use technology to solve it. In most cases, you won't need "exact answers;" calculator numbers will be good enough.

Simplification and Calculator Numbers

When you were in tenth grade, your math teacher may have impressed you with the need to simplify your answers. I'm here to tell you – she was wrong. The form your answer should be in depends entirely on what you will do with it next. In addition, the process of "simplifying," often messy algebra, can ruin perfectly correct answers. From the teacher's point of view, "simplifying" obscures how a student arrived at his answer, and makes problems harder to grade. Moral: don't spend a lot of extra time simplifying your answer. Leave it as close to how you arrived at it as possible.

When should you simplify?

1. Simplify when it actually makes your life easier. For example, in Chapter 2 it's easier to find a second derivative if you simplify the first derivative.
2. Simplify your answer when you need to match it to an answer in the book. You may need to do some algebra to be sure your answer and the book answer are the same.

When you use your calculator

A calculator is required for this course, and it can be a wonderful tool. However, you should be careful not to rely too strongly on your calculator. Follow these rules of thumb:

1. Estimate your answers. If you expect an answer of about 4, and your calculator says 2500, you've made an error somewhere.
2. Don't round until the very end. Every time you make a calculation with a rounded number, your answer gets a little bit worse.
3. When you answer an applied problem, find a calculator number. It doesn't mean much to suggest that the company should produce $\dfrac{\sqrt{12100}\,(2.4)}{2.5}$ items; it's much more meaningful to report that they should produce about 106 items.
4. When you present your final answer, round it to something that makes sense. If you've found an amount of US money, round it to the nearest cent. If you've computed the number of people, round to the nearest person. If there's no obvious context, show your teacher at least two digits after the decimal place.
5. Occasionally in this course, you will need to find the "exact answer." That means – not a calculator approximation. (You can still use your calculator to check your answer.)