4.5.1 The Metric System

Learning Objective(s)

1 Describe the general relationship between the U.S. customary units and metric units of length, weight/mass, and volume.

2 Define the metric prefixes and use them to perform basic conversions among metric units.

Introduction

Objective 1

In the United States, both the **U.S. customary measurement system** and the **metric system** are used, especially in medical, scientific, and technical fields. In most other countries, the metric system is the primary system of measurement. If you travel to other countries, you will see that road signs list distances in kilometers and milk is sold in liters. People in many countries use words like “kilometer,” “liter,” and “milligram” to measure the length, volume, and weight of different objects. These measurement units are part of the metric system.

Unlike the U.S. customary system of measurement, the metric system is based on 10s. For example, a liter is 10 times larger than a deciliter, and a centigram is 10 times larger than a milligram. This idea of “10” is not present in the U.S. customary system—there are 12 inches in a foot, and 3 feet in a yard…and 5,280 feet in a mile!

So, what if you have to find out how many milligrams are in a decigram? Or, what if you want to convert meters to kilometers? Understanding how the metric system works is a good start.

What is Metric?

The metric system uses units such as **meter**, **liter**, and **gram** to measure length, liquid volume, and mass, just as the U.S. customary system uses feet, quarts, and ounces to measure these.

In addition to the difference in the basic units, the metric system is based on 10s, and different measures for length include kilometer, meter, decimeter, centimeter, and millimeter. Notice that the word “meter” is part of all of these units.

The metric system also applies the idea that units within the system get larger or smaller by a power of 10. This means that a meter is 100 times larger than a centimeter, and a kilogram is 1,000 times heavier than a gram. You will explore this idea a bit later. For now, notice how this idea of “getting bigger or smaller by 10” is very different than the relationship between units in the U.S. customary system, where 3 feet equals 1 yard, and 16 ounces equals 1 pound.

Length, Mass, and Volume

The table below shows the basic units of the metric system. Note that the names of all metric units follow from these three basic units.

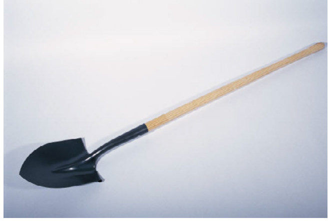
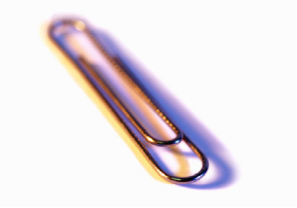
Length	Mass	Volume
basic units		
meter	gram	liter
other units you may see		
kilometer	kilogram	dekaliter
centimeter	centigram	centiliter
millimeter	milligram	milliliter

In the metric system, the basic unit of length is the meter. A meter is slightly larger than a yardstick, or just over three feet.

The basic metric unit of mass is the gram. A regular-sized paperclip has a mass of about 1 gram.

Among scientists, one gram is defined as the mass of water that would fill a 1-centimeter cube. You may notice that the word “mass” is used here instead of “weight.” In the sciences and technical fields, a distinction is made between weight and mass. Weight is a measure of the pull of gravity on an object. For this reason, an object’s weight would be different if it was weighed on Earth or on the moon because of the difference in the gravitational forces. However, the object’s mass would remain the same in both places because mass measures the amount of substance in an object. As long as you are planning on only measuring objects on Earth, you can use mass/weight fairly interchangeably—but it is worth noting that there is a difference!

Finally, the basic metric unit of volume is the liter. A liter is slightly larger than a quart.

		
The handle of a shovel is about 1 meter.	A paperclip weighs about 1 gram.	A medium-sized container of milk is about 1 liter.

Though it is rarely necessary to convert between the customary and metric systems, sometimes it helps to have a mental image of how large or small some units are. The table below shows the relationship between some common units in both systems.

	Common Measurements in Customary and Metric Systems
Length	1 centimeter is a little less than half an inch.
	1.6 kilometers is about 1 mile.
	1 meter is about 3 inches longer than 1 yard.
Mass	1 kilogram is a little more than 2 pounds.
	28 grams is about the same as 1 ounce.
Volume	1 liter is a little more than 1 quart.
	4 liters is a little more than 1 gallon.

Prefixes in the Metric System

Objective **2**

The metric system is a base 10 system. This means that each successive unit is 10 times larger than the previous one.

The names of metric units are formed by adding a prefix to the basic unit of measurement. To tell how large or small a unit is, you look at the **prefix**. To tell whether the unit is measuring length, mass, or volume, you look at the base.

Prefixes in the Metric System						
kilo-	*hecto-*	*deka-*	meter gram liter	*deci-*	*centi-*	*milli-*
1,000 times **larger** than base unit	100 times **larger** than base unit	10 times **larger** than base unit	base units	10 times **smaller** than base unit	100 times **smaller** than base unit	1,000 times **smaller** than base unit

Using this table as a reference, you can see the following:

- A kilogram is 1,000 times larger than one gram (so 1 kilogram = 1,000 grams).
- A centimeter is 100 times smaller than one meter (so 1 meter = 100 centimeters).
- A dekaliter is 10 times larger than one liter (so 1 dekaliter = 10 liters).

Here is a similar table that just shows the metric units of measurement for mass, along with their size relative to 1 gram (the base unit). The common abbreviations for these metric units have been included as well.

Measuring Mass in the Metric System						
kilogram (kg)	hectogram (hg)	dekagram (dag)	gram (g)	decigram (dg)	centigram (cg)	milligram (mg)
1,000 grams	100 grams	10 grams	gram	0.1 gram	0.01 gram	0.001 gram

Since the prefixes remain constant through the metric system, you could create similar charts for length and volume. The prefixes have the same meanings whether they are attached to the units of length (meter), mass (gram), or volume (liter).

Self Check A

Which of the following sets of three units are all metric measurements of length?

A) inch, foot, yard

B) kilometer, centimeter, millimeter

C) kilogram, gram, centigram

D) kilometer, foot, decimeter

Converting Units Up and Down the Metric Scale

Converting between metric units of measure requires knowledge of the metric prefixes and an understanding of the decimal system—that's about it.

For instance, you can figure out how many centigrams are in one dekagram by using the table above. One dekagram is larger than one centigram, so you expect that one dekagram will equal many centigrams.

In the table, each unit is 10 times larger than the one to its immediate right. This means that 1 dekagram = 10 grams; 10 grams = 100 decigrams; and 100 decigrams = 1,000 centigrams. So, 1 dekagram = 1,000 centigrams.

Example

Problem **How many milligrams are in one decigram?**

kg	hg	dag	g	dg	cg	mg
				^		^

Identify locations of milligrams and decigrams.

Decigrams (dg) are larger than milligrams (mg), so you expect there to be many mg in one dg.

					×10	×10
kg	hg	dag	g	dg	cg	mg
				^		^

→ →

Dg is 10 times larger than a cg, and a cg is 10 times larger than a mg.

Since you are going from a larger unit to a smaller unit, multiply.

1 dg • 10 • 10 = 100 mg

Multiply: 1 • 10 • 10, to find the number of milligrams in one decigram.

Answer There are 100 milligrams (mg) in 1 decigram (dg).

Example

Problem **Convert 1 centimeter to kilometers.**

km	hm	dam	m	dm	cm	mm
^					^	

Identify locations of kilometers and centimeters.

Kilometers (km) are larger than centimeters (cm), so you expect there to be less than one km in a cm.

÷10	÷10	÷10	÷10	÷10		
km	hm	dam	m	dm	cm	mm
^					^	

← ← ← ← ←

Cm is 10 times smaller than a dm; a dm is 10 times smaller than a m, etc.

Since you are going from a smaller unit to a larger unit, divide.

1 cm ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 = 0.00001 km

Divide: 1 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10, to find the number of kilometers in one centimeter.

Answer 1 centimeter (cm) = 0.00001 kilometers (km).

Once you begin to understand the metric system, you can use a shortcut to convert among different metric units. The size of metric units increases tenfold as you go up the metric scale. The decimal system works the same way: a tenth is 10 times larger than a hundredth; a hundredth is 10 times larger than a thousandth, etc. By applying what you know about decimals to the metric system, converting among units is as simple as moving decimal points.

Here is the first problem from above: How many milligrams are in one decigram? You can recreate the order of the metric units as shown below:

$$kg \quad hg \quad dag \quad g \quad d\underbrace{g \quad c}_{1}\underbrace{g \quad m}_{2}g$$

This question asks you to start with 1 decigram and convert that to milligrams. As shown above, milligrams is two places to the right of decigrams. You can just move the decimal point two places to the right to convert decigrams to milligrams: $1\,dg = 1\underbrace{0}_{1}\underbrace{0}_{2}.\,mg$.

The same method works when you are converting from a smaller to a larger unit, as in the problem: Convert 1 centimeter to kilometers.

$$k\underbrace{m \quad h}_{5}\underbrace{m \quad d}_{4}\underbrace{am \quad}_{3}\underbrace{m \quad d}_{2}\underbrace{m \quad c}_{1}m \quad mm$$

Note that instead of moving to the right, you are now moving to the left—so the decimal point must do the same: $1\,cm = 0.\underbrace{0}_{5}\underbrace{0}_{4}\underbrace{0}_{3}\underbrace{0}_{2}\underbrace{1}_{1}\,km$.

Self Check B
How many milliliters are in 1 liter?

Self Check C
Convert 3,085 milligrams to grams.

Factor Label Method

There is yet another method that you can use to convert metric measurements—the **factor label method**. You used this method when you were converting measurement units within the U.S. customary system.

The factor label method works the same in the metric system; it relies on the use of unit fractions and the cancelling of intermediate units. The table below shows some of the **unit equivalents** and **unit fractions** for length in the metric system. (You should notice that all of the unit fractions contain a factor of 10. Remember that the metric system is based on the notion that each unit is 10 times larger than the one that came before it.)

Also, notice that two new prefixes have been added here: mega- (which is very big) and micro- (which is very small).

Unit Equivalents	Conversion Factors	
1 meter = 1,000,000 micrometers	$\frac{1\ m}{1{,}000{,}000\ \mu m}$	$\frac{1{,}000{,}000\ \mu m}{1\ m}$
1 meter = 1,000 millimeters	$\frac{1\ m}{1{,}000\ mm}$	$\frac{1{,}000\ mm}{1\ m}$
1 meter = 100 centimeters	$\frac{1\ m}{100\ cm}$	$\frac{100\ cm}{1\ m}$
1 meter = 10 decimeters	$\frac{1\ m}{10\ dm}$	$\frac{10\ dm}{1\ m}$
1 dekameter = 10 meters	$\frac{1\ dam}{10\ m}$	$\frac{10\ m}{1\ dam}$
1 hectometer = 100 meters	$\frac{1\ hm}{100\ m}$	$\frac{100\ m}{1\ hm}$
1 kilometer = 1,000 meters	$\frac{1\ km}{1{,}000\ m}$	$\frac{1{,}000\ m}{1\ km}$
1 megameter = 1,000,000 meters	$\frac{1\ Mm}{1{,}000{,}000\ m}$	$\frac{1{,}000{,}000\ m}{1\ Mm}$

When applying the factor label method in the metric system, be sure to check that you are not skipping over any intermediate units of measurement!

Example

Problem **Convert 7,225 centimeters to meters.**

7,225 cm = ___ m	Meters is larger than centimeters, so you expect your answer to be less than 7,225.
$\frac{7{,}225\ cm}{1} \bullet \frac{1\,m}{100\ cm} = \underline{\quad}\,m$	Using the factor label method, write 7,225 cm as a fraction and use unit fractions to convert it to m.
$\frac{7{,}225\ \cancel{cm}}{1} \bullet \frac{1\,m}{100\ \cancel{cm}} = \underline{\quad}\,m$	Cancel similar units, multiply, and simplify.
$\frac{7{,}225}{1} \bullet \frac{1\,m}{100} = \frac{7{,}225}{100}m$	
$\frac{7{,}225m}{100} = 72.25m$	

Answer 7,225 centimeters = 72.25 meters

Self Check D

Convert 32.5 kilometers to meters.

Now that you have seen how to convert among metric measurements in multiple ways, let's revisit the problem posed earlier.

Example

Problem **If you have a prescription for 5,000 mg of medicine, and upon getting it filled, the dosage reads 5 g of medicine, did the pharmacist make a mistake?**

5,000 mg = ___ g? Need to convert mg to g.

$$\frac{5{,}000\ mg}{1} \bullet \frac{1\,g}{1{,}000\ mg} = \underline{\quad}\,g$$

$$\frac{5{,}000\ \cancel{mg}}{1} \bullet \frac{1\,g}{1{,}000\ \cancel{mg}} = \underline{\quad}\,g$$

$$\frac{5{,}000 \cdot 1\,g}{1 \cdot 1{,}000} = \frac{5{,}000g}{1{,}000}$$

$$\frac{5{,}000g}{1{,}000} = 5\ g$$

Answer 5 g = 5,000 mg, so the pharmacist did not make a mistake.

Summary

The metric system is an alternative system of measurement used in most countries, as well as in the United States. The metric system is based on joining one of a series of prefixes, including kilo-, hecto-, deka-, deci-, centi-, and milli-, with a base unit of measurement, such as meter, liter, or gram. Units in the metric system are all related by a power of 10, which means that each successive unit is 10 times larger than the previous one.

This makes converting one metric measurement to another a straightforward process, and is often as simple as moving a decimal point. It is always important, though, to consider the direction of the conversion. If you are converting a smaller unit to a larger unit, then the decimal point has to move to the left (making your number smaller); if you are converting a larger unit to a smaller unit, then the decimal point has to move to the right (making your number larger).

The factor label method can also be applied to conversions within the metric system. To use the factor label method, you multiply the original measurement by unit fractions; this allows you to represent the original measurement in a different measurement unit.

4.5.1 Self Check Solutions

Self Check A

Which of the following sets of three units are all metric measurements of length?

kilometer, centimeter, millimeter
Correct. All of these measurements are from the metric system. You can tell they are measurements of length because they all contain the word "meter."

Self Check B

How many milliliters are in 1 liter?

There are 10 milliliters in a centiliter, 10 centiliters in a deciliter, and 10 deciliters in a liter. Multiply: 10 • 10 • 10, to find the number of milliliters in a liter, 1,000.

Self Check C

Convert 3,085 milligrams to grams.

One gram is 1,000 times larger than a milligram, so you can move the decimal point in 3,085 three places to the left.

Self Check D

Using whichever method you prefer, convert 32.5 kilometers to meters.

32,500 meters

$\frac{32.5\ km}{1} \bullet \frac{1{,}000\ m}{1\ km} = \frac{32{,}500\ m}{1}$. The km units cancel, leaving the answer in m.

4.5.2 Using Metric Conversions to Solve Problems

Learning Objective(s)
1 Solve application problems involving metric units of length, mass, and volume.

Introduction

Learning how to solve real-world problems using metric conversions is as important as learning how to do the conversions themselves. Mathematicians, scientists, nurses, and even athletes are often confronted with situations where they are presented with information using metric measurements, and must then make informed decisions based on that data.

To solve these problems effectively, you need to understand the context of a problem, perform conversions, and then check the reasonableness of your answer. Do all three of these steps and you will succeed in whatever measurement system you find yourself using.

Understanding Context and Performing Conversions

Objective 1

The first step in solving any real-world problem is to understand its context. This will help you figure out what kinds of solutions are reasonable (and the problem itself may give you clues about what types of conversions are necessary). Here is an example.

Example		
Problem	**Marcus bought at 2 meter board, and cut off a piece 1 meter and 35 cm long. How much board is left?**	
	2 meters – 1 meter 35 cm	To answer this question, we will need to subtract. First convert all measurements to one unit. Here we will convert to centimeters.
	$\frac{2\ m}{1} \bullet \frac{100\ cm}{1\ m} = \_\_\_\ cm$	Use the factor label method and unit fractions to convert from meters to centimeters.
	$\frac{2\ \cancel{m}}{1} \bullet \frac{100cm}{1\ \cancel{m}} = \_\_\_\ cm$	Cancel, multiply, and solve.
	$\frac{200\ cm}{1} = 200\ cm$	

	1 meter + 35 cm 100 cm + 35 cm 135 cm	Convert the 1 meter to centimeters, and combine with the additional 35 centimeters.
	200 cm – 135 cm 65 cm	Subtract the cut length from the original board length.
Answer	There is 65 cm of board left.	

An example with a different context, but still requiring conversions, is shown below.

Example		
Problem	**A faucet drips 10 ml every minute. How much water will be wasted in a week?**	
	$\frac{10\ ml}{1\text{ minute}}\cdot\frac{60\text{ minute}}{1\text{ hour}}\cdot\frac{24\text{ hours}}{1\text{ day}}\cdot\frac{7\text{ days}}{1\text{ week}}$	Start by calculating how much water will be used in a week using the factor label method to convert the time units.
	$\frac{10\ ml}{1\ \cancel{\text{minute}}}\cdot\frac{60\ \cancel{\text{minute}}}{1\ \cancel{\text{hour}}}\cdot\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\cdot\frac{7\ \cancel{\text{days}}}{1\text{ week}}$ $\frac{10\bullet 60\bullet 24\bullet 7\ ml}{1\bullet 1\bullet 1\bullet 1\text{ week}}$	Cancel, multiply and solve.
	$\frac{100800\ ml}{1\text{ week}}$	To give a more useable answer, convert this into liters.
	$\frac{100800\text{ ml}}{1\text{ week}}\bullet\frac{1\text{ L}}{1000\text{ ml}}$ $\frac{100800\ \cancel{\text{ml}}}{1\text{ week}}\bullet\frac{1\text{ L}}{1000\ \cancel{\text{ml}}}$ $\frac{100800\text{ L}}{1000\text{ week}}=100.8\frac{\text{L}}{\text{week}}$	Cancel, multiply and solve.
Answer	The faucet wastes about 100.8 liters each week.	

This problem asked for the difference between two quantities. The easiest way to find this is to convert one quantity so that both quantities are measured in the same unit, and then subtract one from the other.

Self Check A

A bread recipe calls for 600 g of flour. How many kilograms of flour would you need to make 5 loaves?

Checking your Conversions

Sometimes it is a good idea to check your conversions using a second method. This usually helps you catch any errors that you may make, such as using the wrong unit fractions or moving the decimal point the wrong way.

Example

Problem **A bottle contains 1.5 liters of a beverage. How many 250 mL servings can be made from that bottle?**

$1.5\text{ L} \div 250\text{ mL}$	To answer the question, you will need to divide 1.5 liters by 250 milliliters. To do this, convert both to the same unit. You could convert either measurement.
$250\text{ mL} = \_\_\_\text{ L}$	Convert 250 mL to liters
$\frac{250\text{ mL}}{1} \cdot \frac{1\text{L}}{1000\text{ mL}} = \_\_\_\text{ L}$	
$\frac{250\text{ L}}{1000} = 0.25\text{ L}$	
$1.5\text{ L} \div 250\text{ mL} = \frac{1.5\text{ L}}{250\text{ mL}} = \frac{1.5\text{ L}}{0.25\text{ L}}$	Now we can divide using the converted measurement
$\frac{1.5\text{ L}}{0.25\text{ L}} = 6$	

Answer The bottle holds 6 servings.

Having come up with the answer, you could also check your conversions using the quicker "move the decimal" method, shown below.

Example

Problem **A bottle contains 1.5 liters of a beverage. How many 250 mL servings can be made from that bottle?**

$250 \text{ mL} = \_\_\_ \text{ L}$ You need to convert 250 mL to liters

			÷10	÷10	÷10	
kL	hL	daL	L	dL	cL	mL
			^			^
			←	←	←	

On the chart, L is three places to the left of mL.

250. mL → 0.250 L

Move the decimal point three places to the left in 250 mL

$$1.5 \text{ L} \div 250 \text{ mL} = \frac{1.5\,\text{L}}{250\,\text{mL}} = \frac{1.5\,\text{L}}{0.25\,\text{L}}$$

Now divide as was done in the last example

$$\frac{1.5\,\text{L}}{0.25\,\text{L}} = 6$$

Answer The bottle holds 6 servings.

The initial answer checks out — the bottle holds 6 servings. Checking one conversion with another method is a good practice for catching any errors in scale.

Summary

Understanding the context of real-life application problems is important. Look for words within the problem that help you identify what operations are needed, and then apply the correct unit conversions. Checking your final answer by using another conversion method (such as the "move the decimal" method, if you have used the factor label method to solve the problem) can cut down on errors in your calculations.

4.5.2 Self Check Solutions

Self Check A

A bread recipe calls for 600 g of flour. How many kilograms of flour would you need to make 5 loaves?

Multiplying 600 g per loaf by the 5 loaves,
$600\text{g} \cdot 5 = 3000 \text{ g}$
Using factor labels or the "move the decimal" method, convert this to 3 kilograms
You will need 3 kg of flour to make 5 loaves.