3.5 Solving Percent Problems

Learning Objective(s)
1 Identify the amount, the base, and the percent in a percent problem.
2 Find the unknown in a percent problem.

Introduction

Percents are a ratio of a number and 100. So they are easier to compare than fractions, as they always have the same denominator, 100. A store may have a 10% off sale. The amount saved is always the same portion or fraction of the price, but a higher price means more money is taken off. Interest rates on a saving account work in the same way. The more money you put in your account, the more money you get in interest. It's helpful to understand how these percents are calculated. In the last section we found a percent of a whole; in this section we will expand on that.

Parts of a Percent Problem

Objective 1

Jeff has a coupon at the Guitar Store for 15% off any purchase of $100 or more. He wants to buy a used guitar that has a price tag of $220 on it. Jeff wonders how much money the coupon will take off the original $220 price.

Problems involving percents have any three quantities to work with: the **percent**, the **amount**, and the **base**.

The percent has the percent symbol (%) or the word "percent." In the problem above, 15% is the percent off the purchase price.

The base is the whole amount. In the problem above, the whole price of the guitar is $220, which is the base.

The amount is the number that relates to the percent. It is always part of the whole. In the problem above, the amount is unknown. Since the percent is the percent *off*, the amount will be the *amount off* of the price.

You will return to this problem a bit later. The following examples show how to identify the three parts, the percent, the base, and the amount.

Example	
Problem	**Identify the percent, amount, and base in this problem.** **30 is 20% of what number?**
Percent: The percent is the number with the % symbol: **20%**. **Base:** The base is the whole amount, which in this case is unknown. **Amount:** The amount based on the percent is **30**.	
Answer	Percent = 20% Amount = 30 Base = unknown

The previous problem states that 30 is a portion of another number. That means 30 is the amount. Note that this problem could be rewritten: 20% of what number is 30?

Example	
Problem	**Identify the percent, amount, and base in this problem.** **What percent of 30 is 3?**
Percent: The percent is unknown, because the problem states "what percent?". **Base:** The base is the whole amount, so the base is 30. **Amount:** The amount is a portion of the whole, which is 3 in this case..	
Answer	Percent = unknown Amount = 3 Base = 30

Solving with Equations

Objective 2

Percent problems can be solved by writing **equations.** An equation uses an equal sign (=) to show that two mathematical expressions have the same value.

Percents are fractions, and just like fractions, when finding a percent (or fraction, or portion) of another amount, you multiply.

The percent of the base is the amount.

The Percent Equation

Percent **of the** Base **is the** Amount.
Percent • Base = Amount

In the examples below, the unknown is represented by the letter *n.* The unknown can be represented by any letter or a box ☐ or even a question mark.

Example

Problem **Write an equation that represents the following problem. 30 is 20% of what number?**

20% of what number is 30?	Rewrite the problem in the form "percent of base is amount."
Percent is: 20% Base is: unknown Amount is: 30	Identify the percent, the base, and the amount.
Percent • Base = Amount 20% • *n* = 30	Write the percent equation. using *n* for the base, which is the unknown value.

Answer 20% • *n* = 30.

Once you have an equation, you can solve it and find the unknown value. To do this, think about the relationship between multiplication and division. Look at the pairs of multiplication and division facts below, and look for a pattern in each row.

Multiplication	Division
2 • 3 = 6	6 ÷ 2 = 3
8 • 5 = 40	40 ÷ 8 = 5
7 • 4 = 28	28 ÷ 7 = 4
6 • 9 = 54	54 ÷ 6 = 9

Multiplication and division are inverse operations. What one does to a number, the other "undoes."

When you have an equation such as 20% • *n* = 30, you can divide 30 by 20% to find the unknown: *n* = 30 ÷ 20%.

You can solve this by writing the percent as a decimal or fraction and then dividing.

$$n = 30 \div 20\% = 30 \div 0.20 = 150$$

Example		
Problem	**What percent of 72 is 9?**	
	Percent: unknown Base: 72 Amount: 9	Identify the percent, base, and amount.
	$n \bullet 72 = 9$	Write the percent equation: Percent • Base = Amount. Use n for the unknown (percent).
	$n = 9 \div 72$	Divide to undo the multiplication of n times 72.
	$72\overline{)9.000}$ 0.125	Divide 9 by 72 to find the value for n, the unknown.
	$n = 0.125$	Move the decimal point two places to the right to write the decimal as a percent.
	$n = 12.5\%$	
Answer	12.5% of 72 is 9.	

You can estimate to see if the answer is reasonable. Use 10% and 20%, numbers close to 12.5%, to see if they get you close to the answer.

10% of 72 = 0.1 • 72 = 7.2
20% of 72 = 0.2 • 72 = 14.4

Notice that 9 is between 7.2 and 14.4, so 12.5% is reasonable since it is between 10% and 20%.

Example		
Problem	**What is 110% of 24?**	
	Percent: 110% Base: 24 Amount: unknown	Identify the percent, the base, and the amount.
	110% • 24 = n	Write the percent equation. Percent • Base = Amount. The amount is unknown, so use n.
	1.10 • 24 = n	Write the percent as a decimal by moving the decimal point two places to the left.
	1.10 • 24 = 26.4 = n	Multiply 24 by 1.10 or 1.1.
Answer	26.4 is 110% of 24.	

This problem is a little easier to estimate. 100% of 24 is 24. And 110% is a little bit more than 24. So, 26.4 is a reasonable answer.

Self Check A

18 is what percent of 48?

Self Check B

18 is 125% of what number?

Summary

Percent problems have three parts: the percent, the base (or whole), and the amount. Any of those parts may be the unknown value to be found. To solve percent problems, you can use the equation, Percent • Base = Amount, and solve for the unknown numbers.

3.5 Self Check Solutions

Self Check A

18 is what percent of 48?

The equation for this problem is $n \bullet 48 = 18$. The corresponding division is $18 \div 48$, so $n = 0.375$. Rewriting this decimal as a percent gives 37.5%.

Self Check B

18 is 125% of what number?

125% written as a decimal is 1.25.
The equation for this problem is $n \bullet 1.25 = 18$. Dividing $18 \div 1.25$ gives $n = 14.4$.