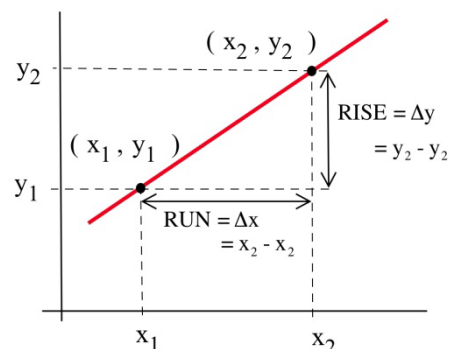


Chapter 2: The Derivative

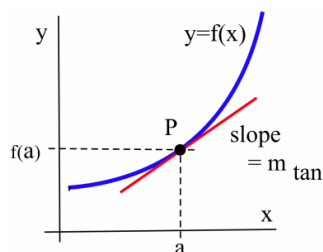
Precalculus Idea: Slope and Rate of Change

The slope of a line measures how fast a line rises or falls as we move from left to right along the line. It measures the rate of change of the y -coordinate with respect to changes in the x -coordinate. If the line represents the distance traveled over time, for example, then its slope represents the velocity. In the figure, you can remind yourself of how we calculate slope using two points on the line:

$$m = \text{slope from P to Q} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$



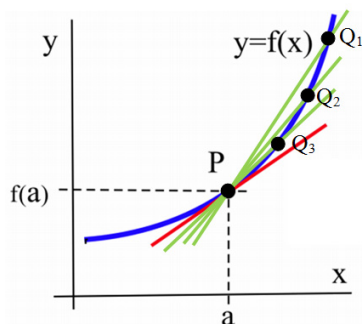
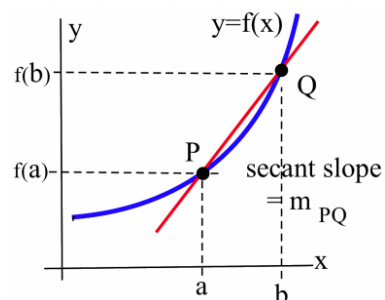
We would like to be able to get that same sort of information (how fast the curve rises or falls, velocity from distance) even if the graph is not a straight line. But what happens if we try to find the slope of a curve at a point, as in the figure below? We need two points in order to determine the slope of a line. How can we find a slope of a curve, at just one point?



The answer, as suggested in the figure, is to find the slope of the tangent line to the curve at that point. Most of us have an intuitive idea of what a tangent line is. Unfortunately, “tangent line” is hard to define precisely.

Definition: A **secant line** is a line between two points on a curve, as in the figure to the right.

Can't-quite-do-it-yet Definition: A **tangent line** is a line at one point on a curve that does its best to be the curve at that point?



As you may be able to see, the closer the point Q is to the point P , the closer the secant slope gets to the tangent slope. This will be key to finding the tangent slope, but first we need to more carefully define the idea of “getting closer to.”