Section 4: Exponents

The Laws of Exponents let you rewrite algebraic expressions that involve exponents. The last three listed here are really definitions rather than rules.

Laws of Exponents:
All variables here represent real numbers and all variables in denominators are nonzero.

1) $x^a \cdot x^b = x^{a+b}$ 2) $\frac{x^a}{x^b} = x^{a-b}$ 3) $\left(x^a\right)^b = x^{ab}$

4) $(xy)^a = x^a y^a$ 5) $\left(\frac{x}{y}\right)^b = \frac{x^b}{y^b}$ 6) $x^0 = 1$, provided $x \neq 0$

7) $x^{-n} = \frac{1}{x^n}$, provided $x \neq 0$ 8) $x^{1/n} = \sqrt[n]{x}$, provided $x \neq 0$

Example 1

Simplify $\left(2x^2\right)^3(4x)$.

We'll begin by simplifying the $\left(2x^2\right)^3$ portion. Using property #4, we can write

$2^3\left(x^2\right)^3(4x)$ Evaluate 2^3, and use property #3

$8x^6(4x)$ Multiply the constants, and use property #1, recalling $x = x^1$

$32x^7$

Being able to work with negative and fractional exponents will be very important later in this course.

Example 2

Rewrite $\frac{5}{x^3}$ using negative exponents.

Since $x^{-n} = \frac{1}{x^n}$, then $x^{-3} = \frac{1}{x^3}$.

$\frac{5}{x^3} = 5x^{-3}$

Example 3

Simplify as much as possible and write your answer using only positive exponents: $\left(\frac{x^{-2}}{y^{-3}}\right)^2$

$\left(\frac{x^{-2}}{y^{-3}}\right)^2 = \frac{\left(x^{-2}\right)^2}{\left(y^{-3}\right)^2} = \frac{x^{-4}}{y^{-6}} = \frac{y^6}{x^4}$

Example 4

Rewrite $4\sqrt{x} - \frac{3}{\sqrt{x}}$ using exponents.

A square root is a radical with index of two. In other words, $\sqrt{x} = \sqrt[2]{x}$. Using the exponent rule give above, $\sqrt{x} = \sqrt[2]{x} = x^{1/2}$. Rewriting the square roots using the fractional exponent,

$4\sqrt{x} - \frac{3}{\sqrt{x}} = 4x^{1/2} - \frac{3}{x^{1/2}}$

Now we can use the negative exponent rule to rewrite the second term in the expression.

$4x^{1/2} - \frac{3}{x^{1/2}} = 4x^{1/2} - 3x^{-1/2}$

Example 5

Rewrite using only positive exponents: $\left(\sqrt{p^5}\right)^{-1/3}$

$\left(\sqrt{p^5}\right)^{-1/3} = \left(\left(p^5\right)^{1/2}\right)^{-1/3} = p^{-5/6} = \frac{1}{p^{5/6}}$

Example 6

Rewrite $x^{-4/3}$ as a radical.

First, we can use the negative exponent rule to rewrite this as $\frac{1}{x^{4/3}}$.

Since $\frac{4}{3} = 4 \cdot \frac{1}{3}$, we can use laws of exponents to rewrite this as $\frac{1}{\left(x^{1/3}\right)^4}$.

Now using the radical equivalence, we can rewrite this as $\frac{1}{\left(\sqrt[3]{x}\right)^4}$.

1.4 Exercises

Simplify each expression

1. x^3x^5
2. x^4x^2
3. $\left(x^3\right)^4$
4. $\left(x^7\right)^2$
5. $\left(2x^2\right)^3 x^4$
6. $\left(5x^4\right)^2 x^5$
7. $\dfrac{\left(3x^2\right)^2}{6x^3}$
8. $\dfrac{5x\left(4x\right)^2}{2x^2}$

Simplify, and rewrite without negative exponents

9. $4x^{-3}$
10. $2x^{-5}$
11. $x^{-4}x^2$
12. $x^{-2}x$
13. $\dfrac{5x^{-3}}{2x^{-6}}$
14. $\dfrac{2x^{-4}}{6x^{-2}}$

Rewrite using negative or fractional exponents

15. $\dfrac{4}{x^{-5}}$
16. $\dfrac{4}{x^{-3}}$
17. $3\sqrt{x}$
18. $\sqrt[4]{x}$
19. $\dfrac{4}{\sqrt[3]{x}}$
20. $\dfrac{1}{5\sqrt{x}}$

Rewrite as a radical

21. $4x^{-1/2}$
22. $5x^{-1/3}$
23. $2x^{1/3}$
24. $5x^{3/2}$